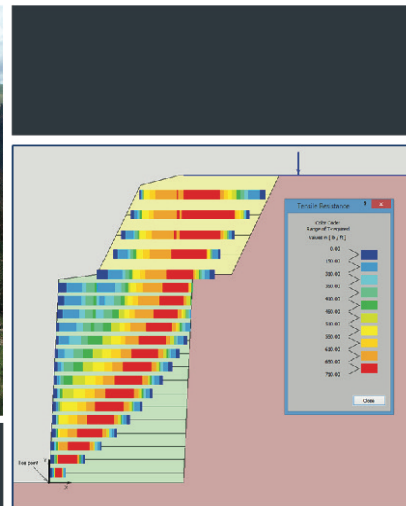


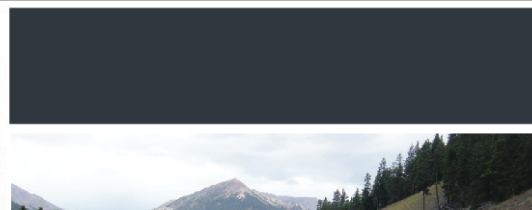
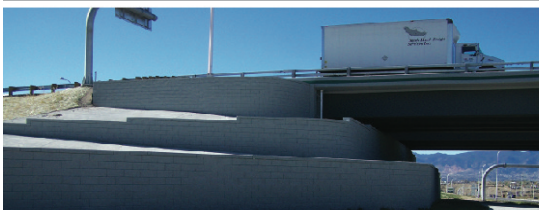
Robust Design Framework for Geosynthetic Reinforced Walls and Slopes



Dov Leshchinsky
Emeritus Professor, *University of Delaware*
Co-Founder, *ADAMA Engineering, Oregon*



LIMIT EQUILIBRIUM DESIGN FRAMEWORK FOR MSE STRUCTURES WITH EXTENSIBLE REINFORCEMENT



- 1. Provides analytical details**
- 2. Verifies using numerical and physical models**

Technical Report Documentation Page

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9. Performing Organization Name and Address Parsons Brinckerhoff 1015 Half Street, SE, Suite 650 Washington, DC 20003 ¹ ADAMA Engineering, Inc., 12042 SE Sunnyside Rd., Suite 711, Clackamas, OR 97015		10. Work Unit No. (TRAIS)	
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16. Abstract Current design of reinforced soil structures in the U.S. distinguishes between slopes and walls using the batter angle as a criterion. Using a unified approach in limit state design of reinforced 'walls' and 'slopes' should diminish confusion while enabling a wide and consistent usage in solving geotechnical problems such as complex geometries and soil profiles. Limit equilibrium (LE) analysis has been used successfully in the design of complex and critical (e.g., tall dams) for many decades. Limit state analysis, including LE, assumes that the <i>design</i> strength of the soil is mobilized. Presented is a LE framework, limited to extensible reinforcement, which enables the designer to find the tensile force distribution in each layer required at a limit state. This approach is restricted to Allowable Stress Design (ASD). Three example problems are presented.			
17. Key Words Mechanically Stabilized Earth Wall Design, MSE Wall Design, Limit Equilibrium, Geotechnical, Extensible reinforcement		18. Distribution Statement No restrictions.	
19. Security Classif. (of this report) UNCLASSIFIED	20. Security Classif. (of this UNCLASSIFIED	21. No. of Pages 120	22. Price

Roadmap of Presentation

- **Why Limit State analysis is needed?**
- Available Limit State Methods of Analysis
- Limit Equilibrium: Global Approach
- The Safety Map Tool
- Limit Equilibrium: Baseline Solution
- Limit Equilibrium: Design Approach
- Limit Equilibrium: Examples
- Concluding Remarks

Why Limit State Analysis is Needed?

- **Collapse is a realistic possibility**
- Such limit state is avoided by assigning adequate margins of safety in design
- **To quantify margins of safety, one needs to reliably predict limit state conditions**
- Theoretically, design considering serviceability alone should eliminate possible limit state
- **However, predicting displacements in practice is poor whereas predicting failure is quite reliable**
- ∴ **Practice:** Design for limit state using adequate margins of safety implicitly satisfies serviceability

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Limit State: *Analysis or Design*

- Premise of Limit State *Analysis*:
 - Failure is imminent
 - Strength of all elements resisting failure are mobilized simultaneously
- Premise of Limit State *Design*:
 - Developed 'active wedge' is resisted by reinforcement → Select reinforcement with adequate margin of safety against rupture
 - Ensure existence of margin of safety on strength of soil
- *Analysis* is the basis for *Design* → It defines conditions for imminent failure thus it allows meaningful use of prescribed margins of safety on strengths

Synergistic Approach: Layout & Strength

- **Internal Stability**

- Strength, Connection, Pullout
- Analyses Determine Strength and Length

- **External Stability**

- Bearing Capacity, Direct Sliding, Eccentricity (Overturning)
- Analyses Determine Length

- **Global/Compound**

- Slope Stability Analysis
- Analysis Determines Strength and Length

- **Do we need such disjointed analyses?**

Limit State Analysis: Lateral Earth Pressure - Simplified AASHTO (Internal Stability)

- Semi-empirical, calibrated at working load conditions (i.e., not at limit state)
- Safe, fortunately economical, and easy to use →
Credit: Turned an innovative technology into a commodity
- Batter is limited to $\leq 20^\circ$ → What about slopes?
- What about complex geometries? Extrapolation to realistic geotechnical conditions (e.g., variable layout of reinforcement, marginal soils)?
- Is it actually adequate for limit state? If not, could it be overly-conservative?

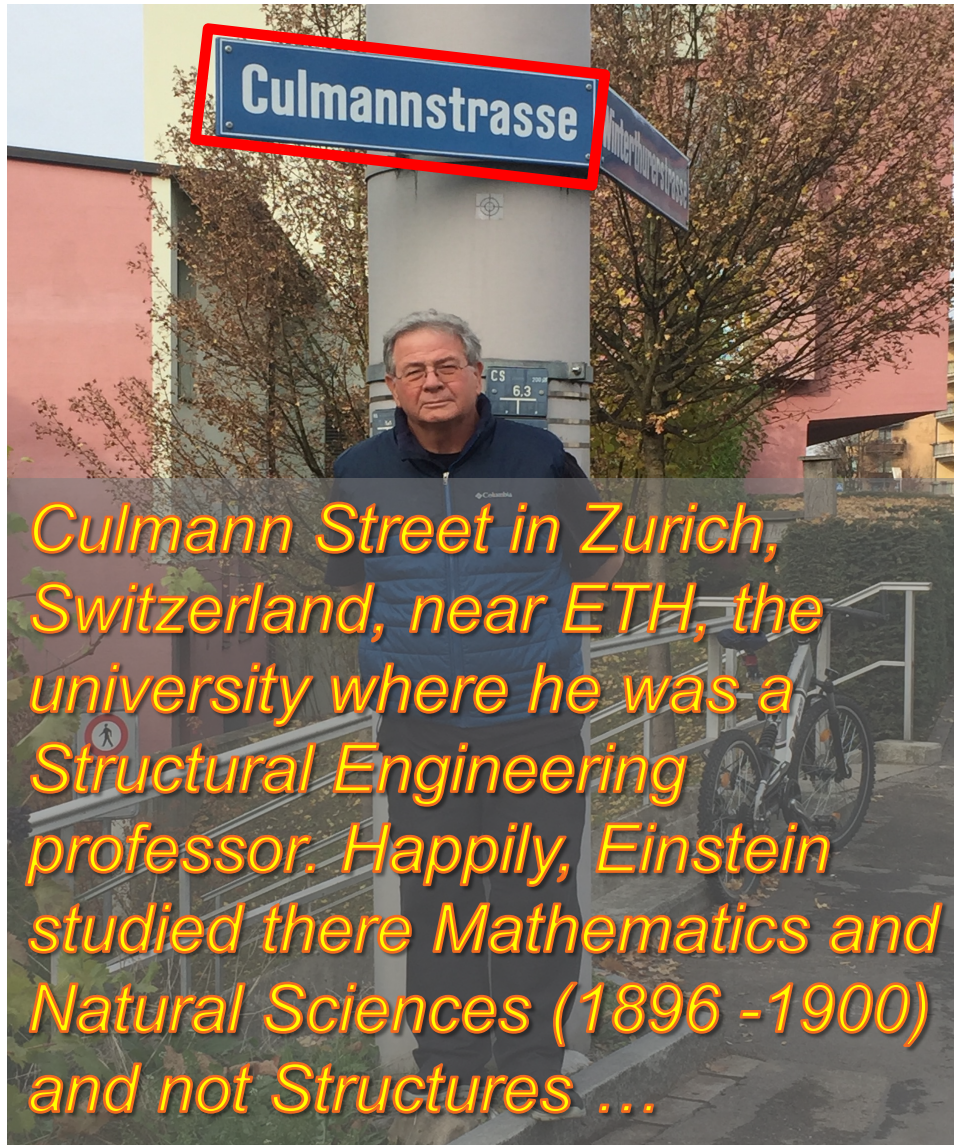
Limit State Analysis: Continuum Mechanics (FE, FD)

- Comprehensive approach
- Valid for walls and slopes
- More complicated than AASHTO → Could be useful in identifying potential failure geometries in complex problems
- Not yet a common design tool in the US
- Impractical tool to generate the *instructive* Tension Map at limit state (i.e., baseline solution explained later)

Limit State Analysis: **Global** **Limit Equilibrium (LE)**

- Simple and yet applicable to complex problems
- No arbitrary distinction between 'wall' and 'slope'
- Global LE design is half-cooked → Strength is examined globally - along a singular slip surface - while locally **required** strength, including connections, is overlooked → That is, it ignores local demand by smearing (shedding) the load amongst all layers
- ∴ Does not deal explicitly with 'Internal Stability' which is concerned with local demand → It provides an important, but narrow, design perspective

LE is Classic...

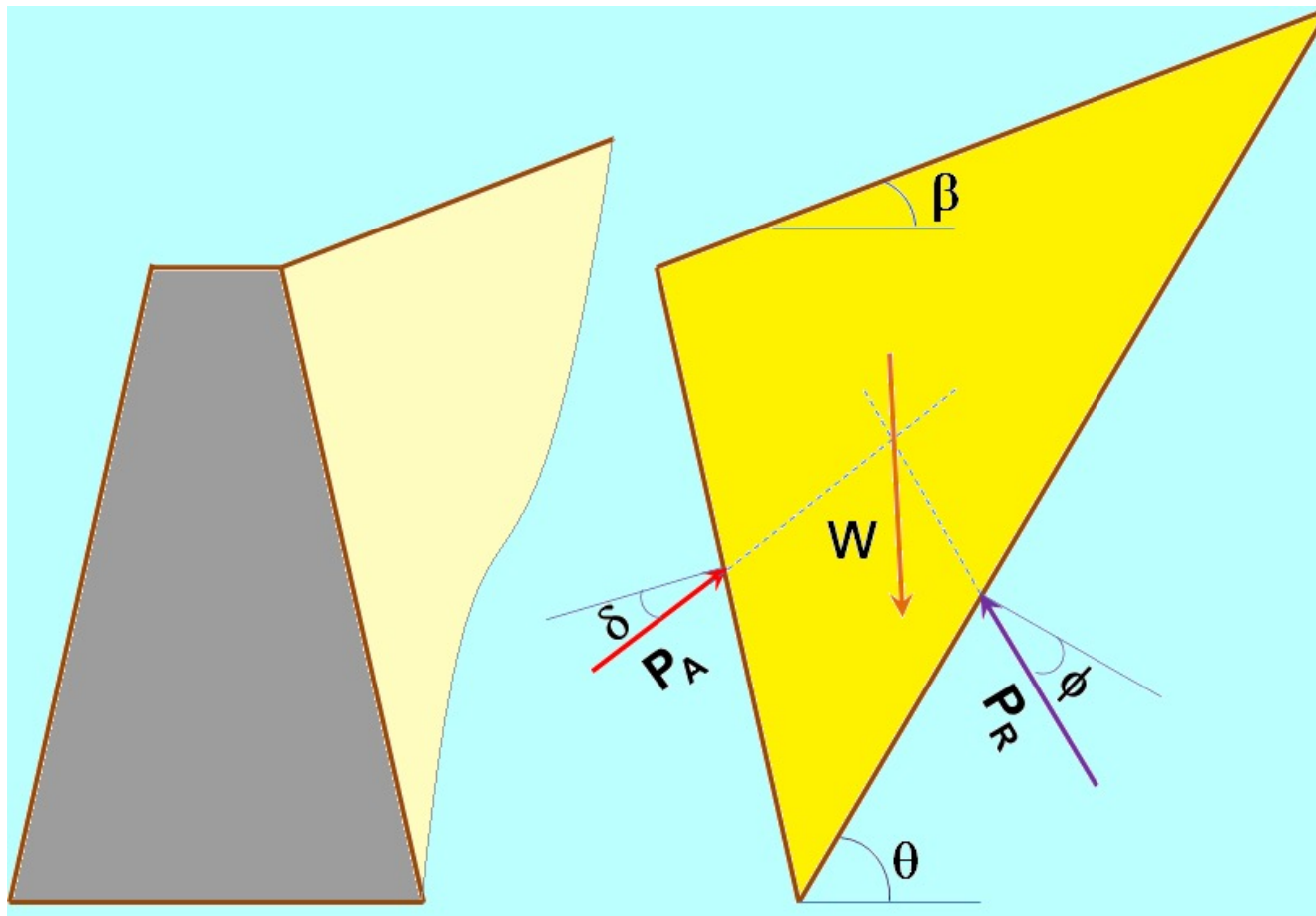


- Coulomb in 1766: Resultant force on a retaining is based on LE approach
- First formulation related to slope stability: Culmann Wedge in 1866

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Coulomb (1776) Active Wedge – Gravity Wall: Find $\max(P_A)$



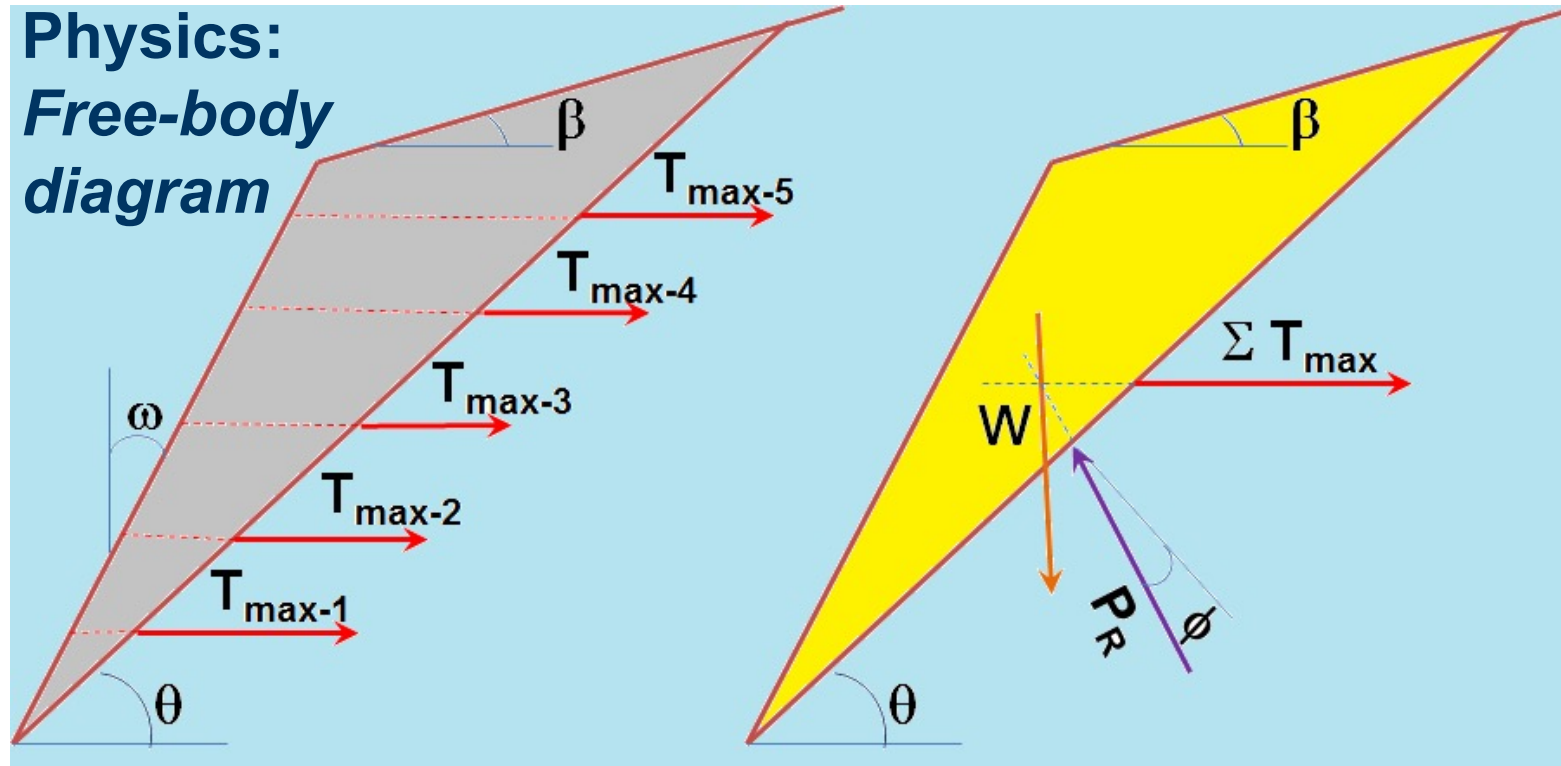
Physics is timeless...

→ Free-body diagram yielding equilibrium

Premise: Small outwards wall movement → Active soil wedge forms → P_o drops to P_A

Note: Formation of planar surface does not mean wall failure
→ Wall is designed to resist the active wedge

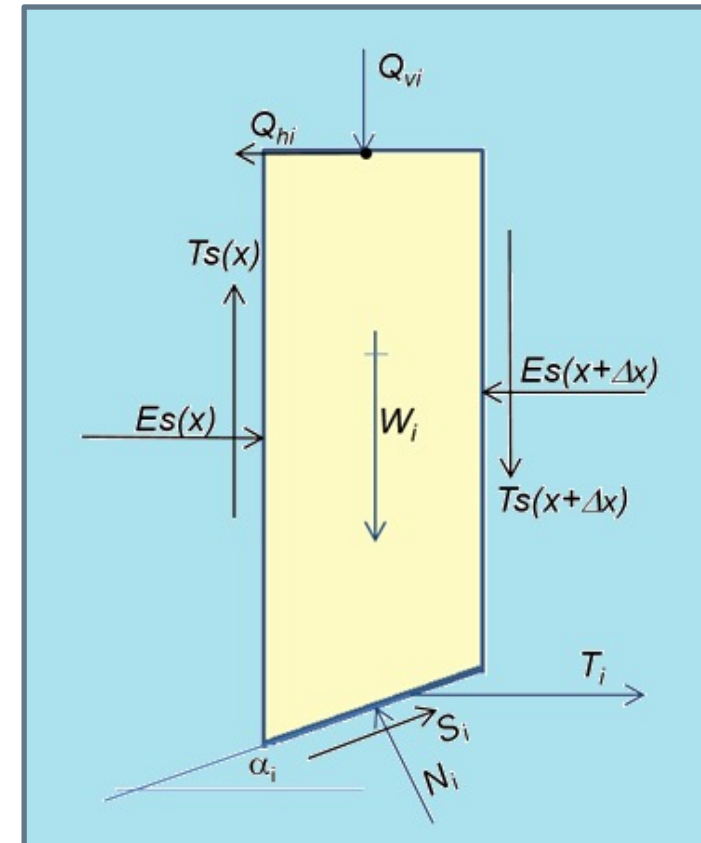
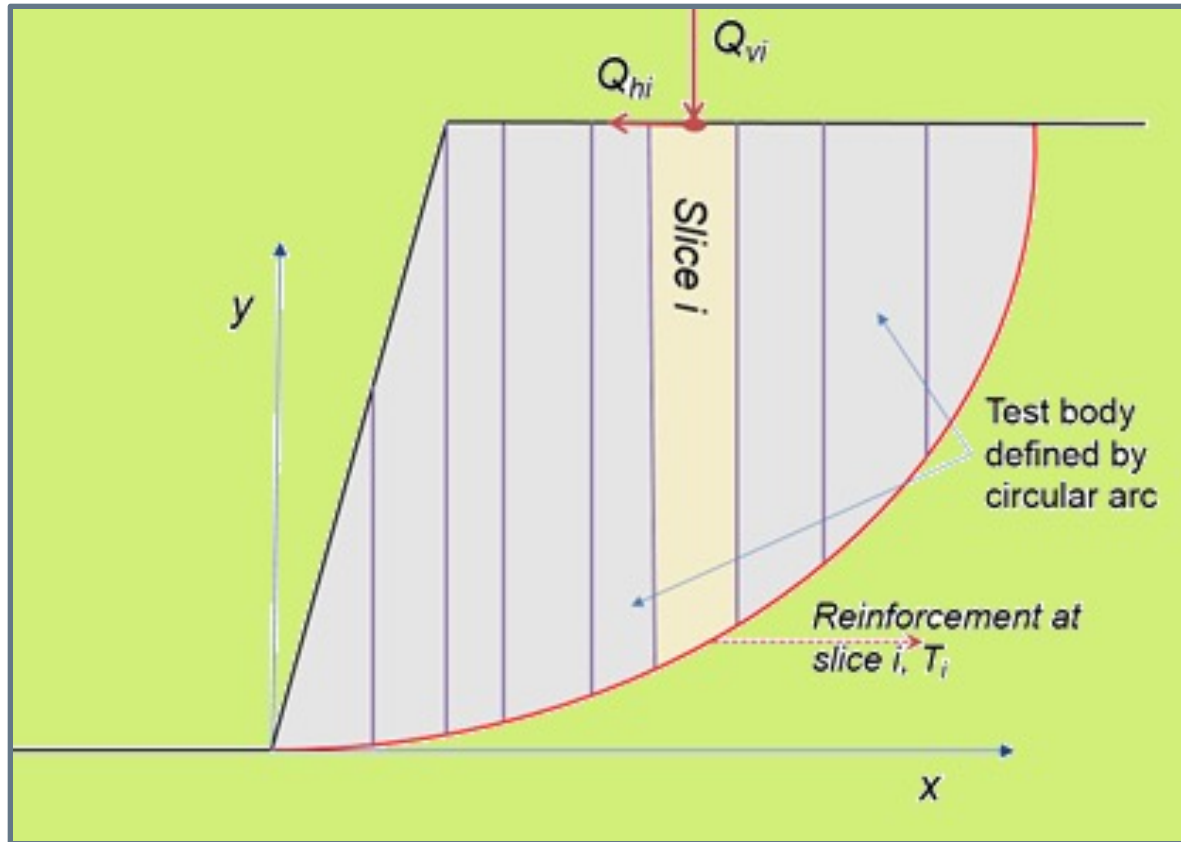
Culmann (1866) Critical Wedge for Reinforced Slope: Find $\max \Sigma(T_{\max})$



Small stretch of reinforcement →
Active wedge develops →
Load in reinforcement drops to T_{\max}

Note: Formation of slip surface does not mean structural failure →
Reinforcement is designed to resist the active soil wedge

Bishop (1955) Circular Arc: Find $\min(SF) = F_s$

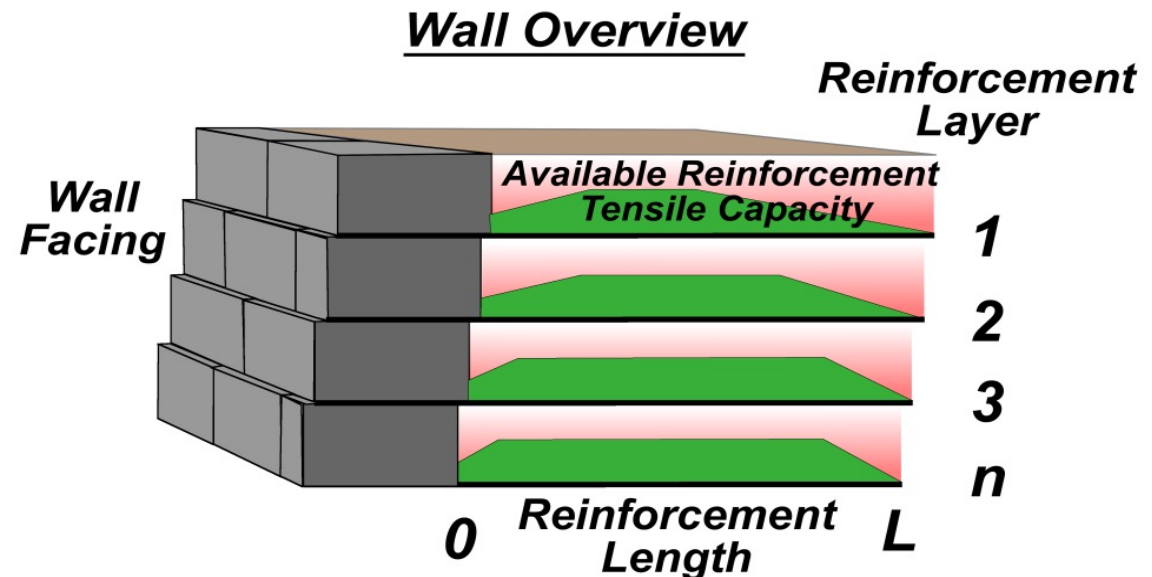
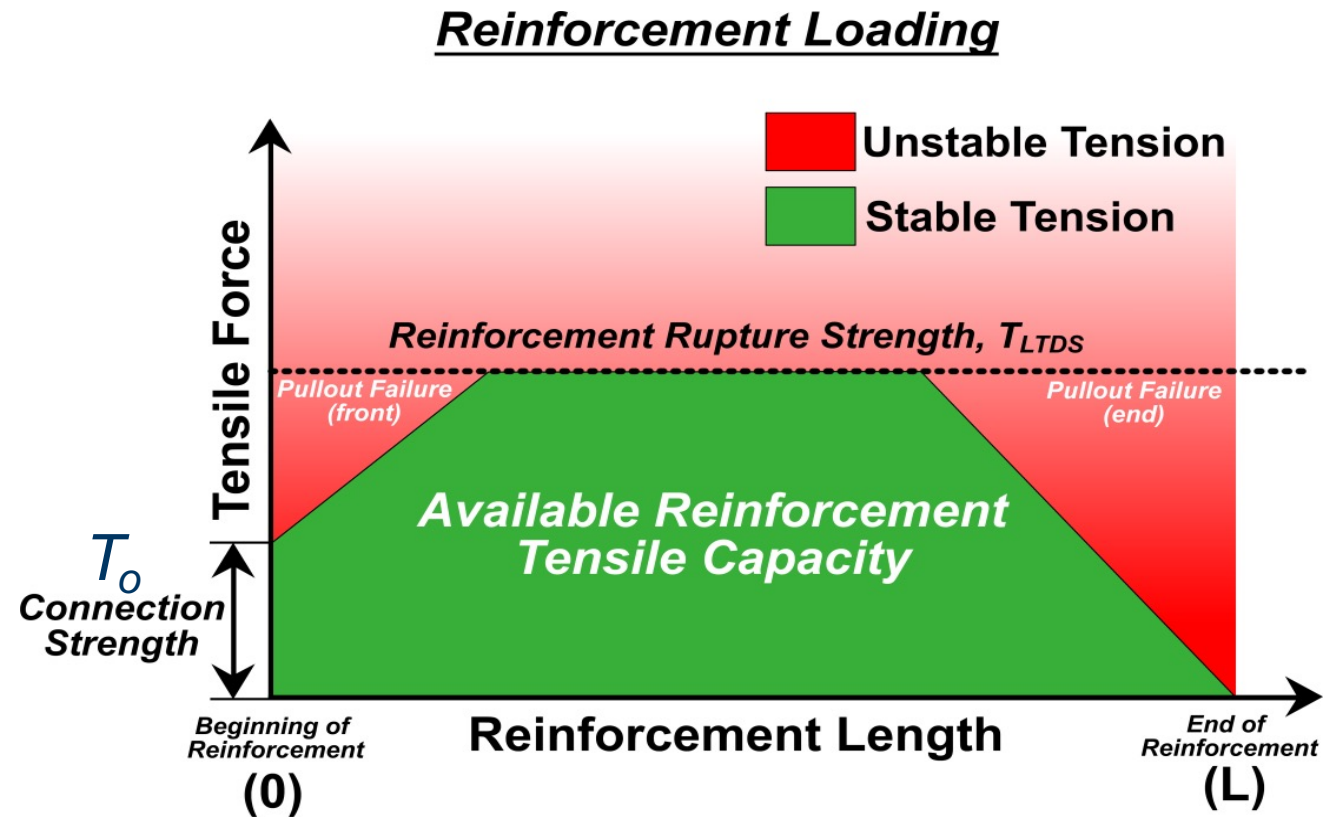


Bishop considers layered soil/complex problems. Circle can degenerate to planar surface (if it is more critical) but **a priori** assumed planar surface cannot degenerate to curved surface → Valid for slopes and walls...

LE:

T_i = Tensile Capacity
Along Each Layer of
Reinforcement

(Note: Front and rear
pullout resistance
enables the
mobilization of
 $LTDS = T_{LTDS}$)



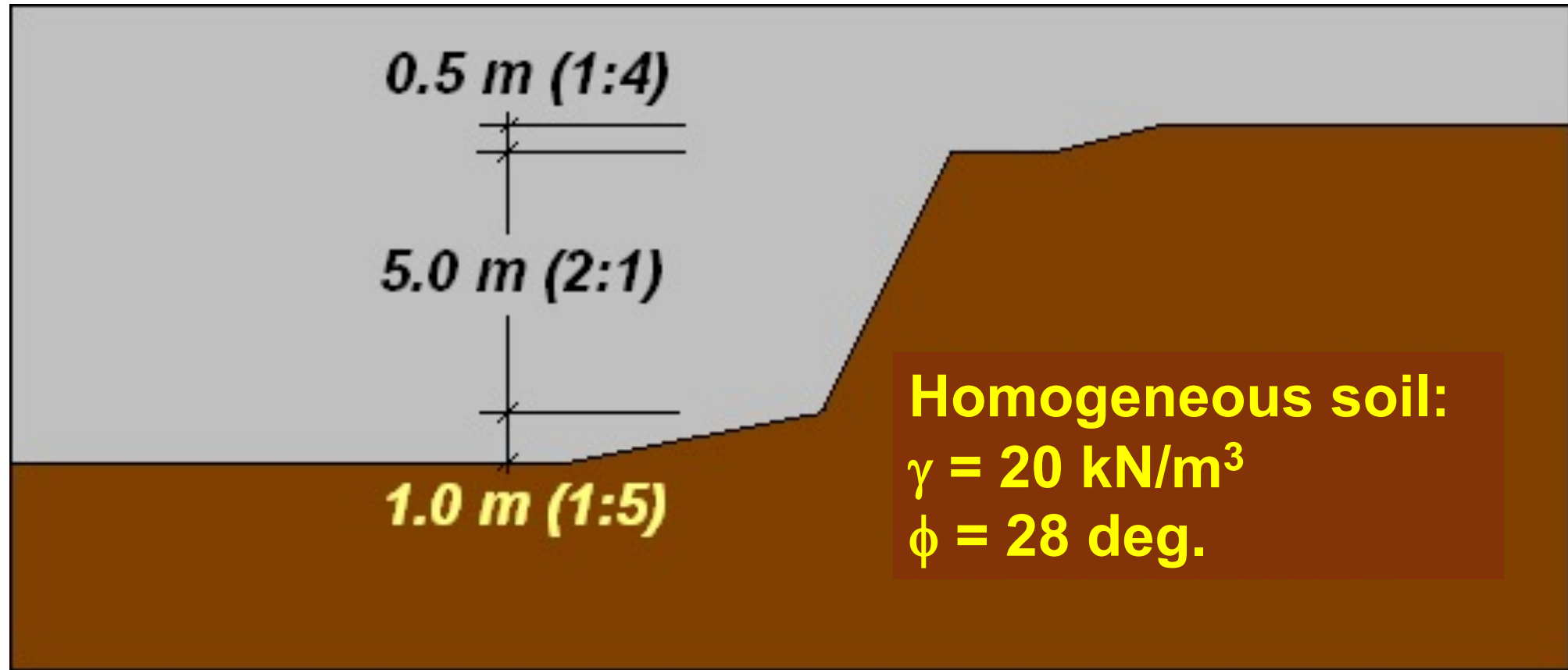
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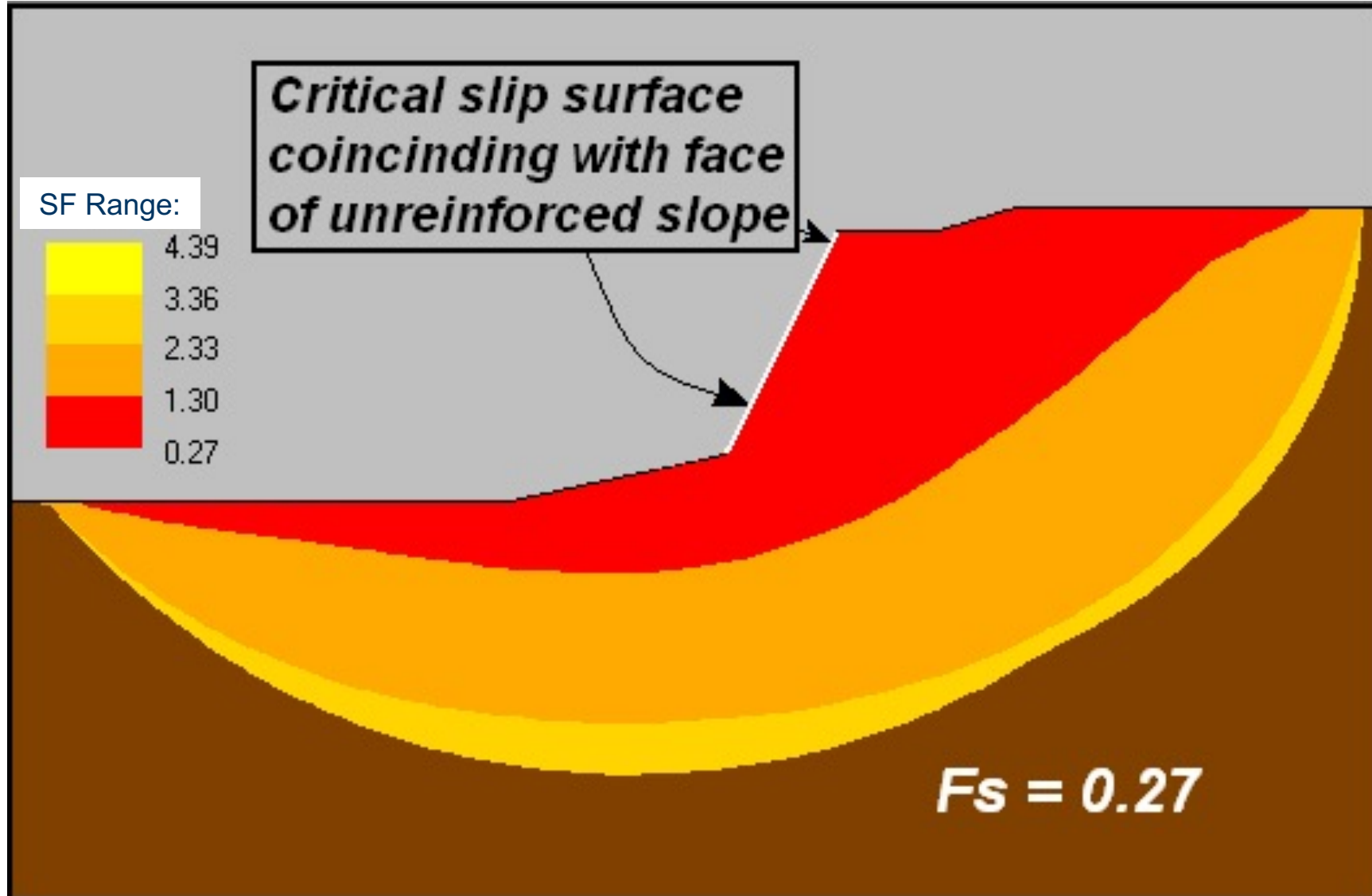
The Safety Map Tool

- **Safety Map:** Baker and Leshchinsky (2001) introduced the concept, proved its mathematical validity, and coined the term
- **Safety Map** = Color-coded map showing the *spatial* distribution of the *safety factors*, SF, in a slope → Visual diagnostic tool for the state of stability of a reinforced mass
- **Design Objective:** Select strength & layout of reinforcement to produce an efficient structure that is adequately stable

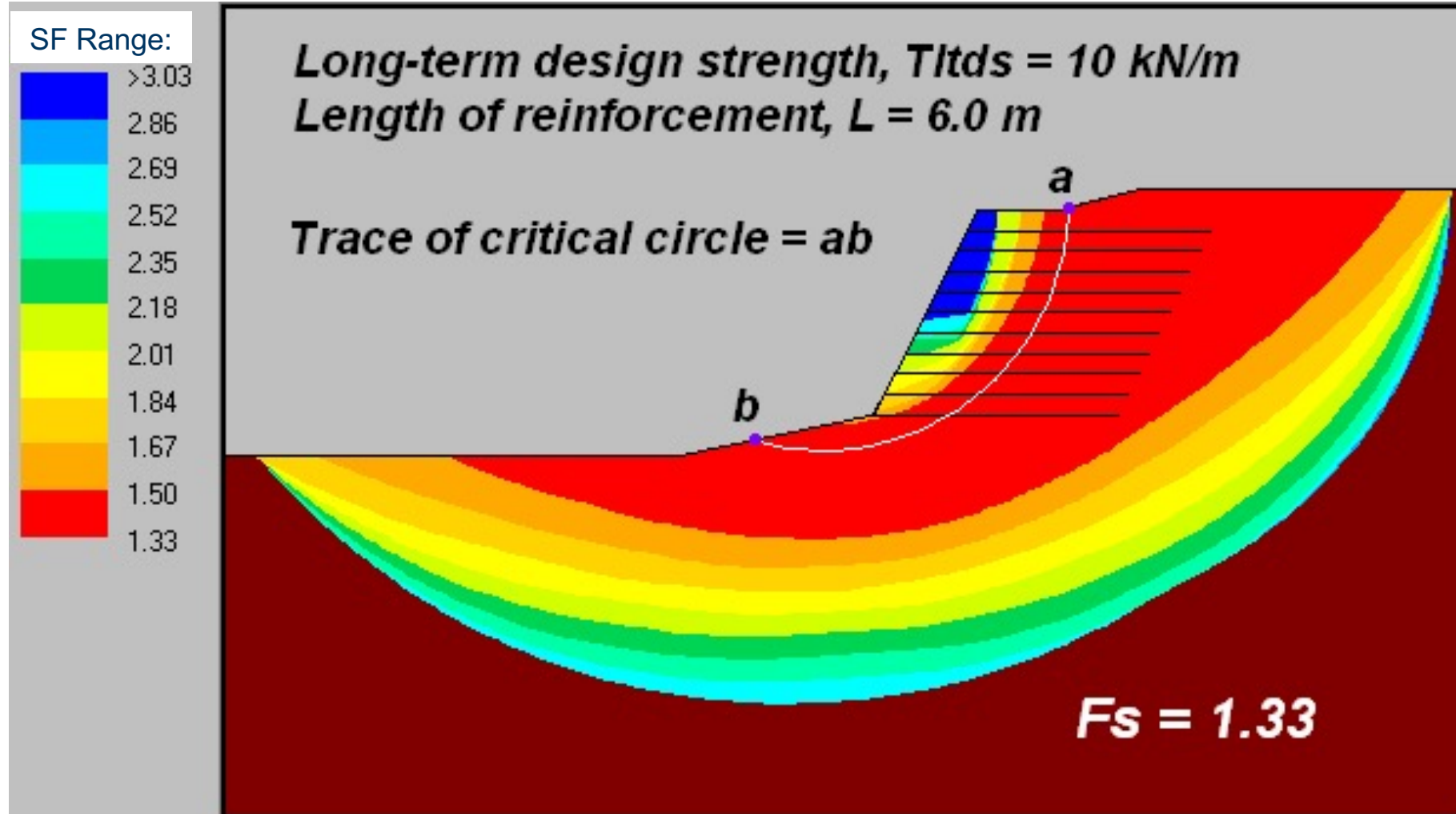
Example Problem



Unreinforced Problem (Bishop)



Adequate Reinforcement Layout using Circular Arc (Bishop)



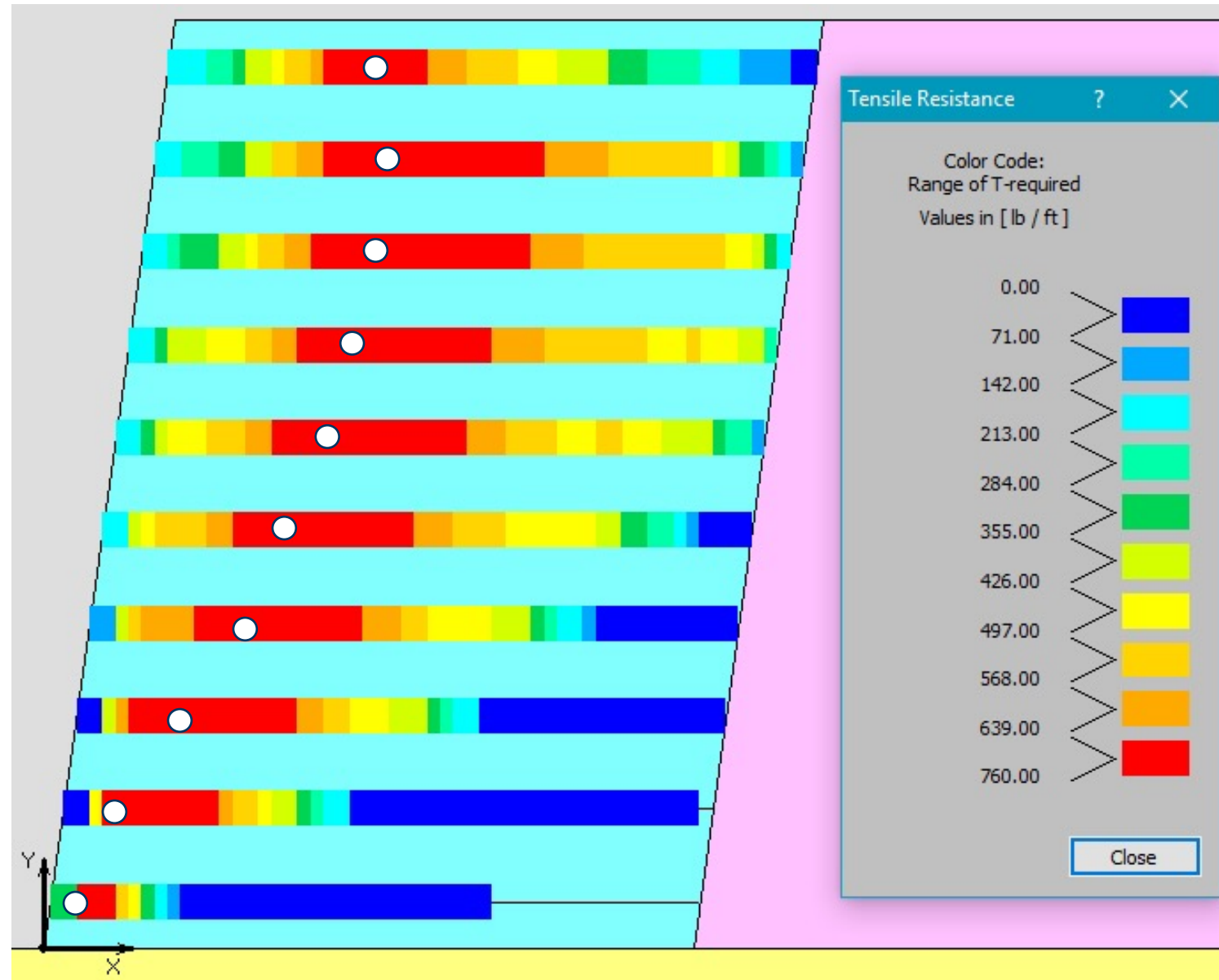
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Inverse of Safety Map...

- **Safety Map** finds the *spatial* distribution of the *safety factors*, SF, in a reinforced soil mass
- Conversely, Internal Stability analysis in LE produces the tensile resistance needed for **Fs=SF=1.0 everywhere**
- The Internal Stability approach produces the *baseline solution*: **Tension Map** means $T_{req}(x)$, including T_{max} and T_0 for each layer → It leads to a rational and robust selection of reinforcement strength and facing

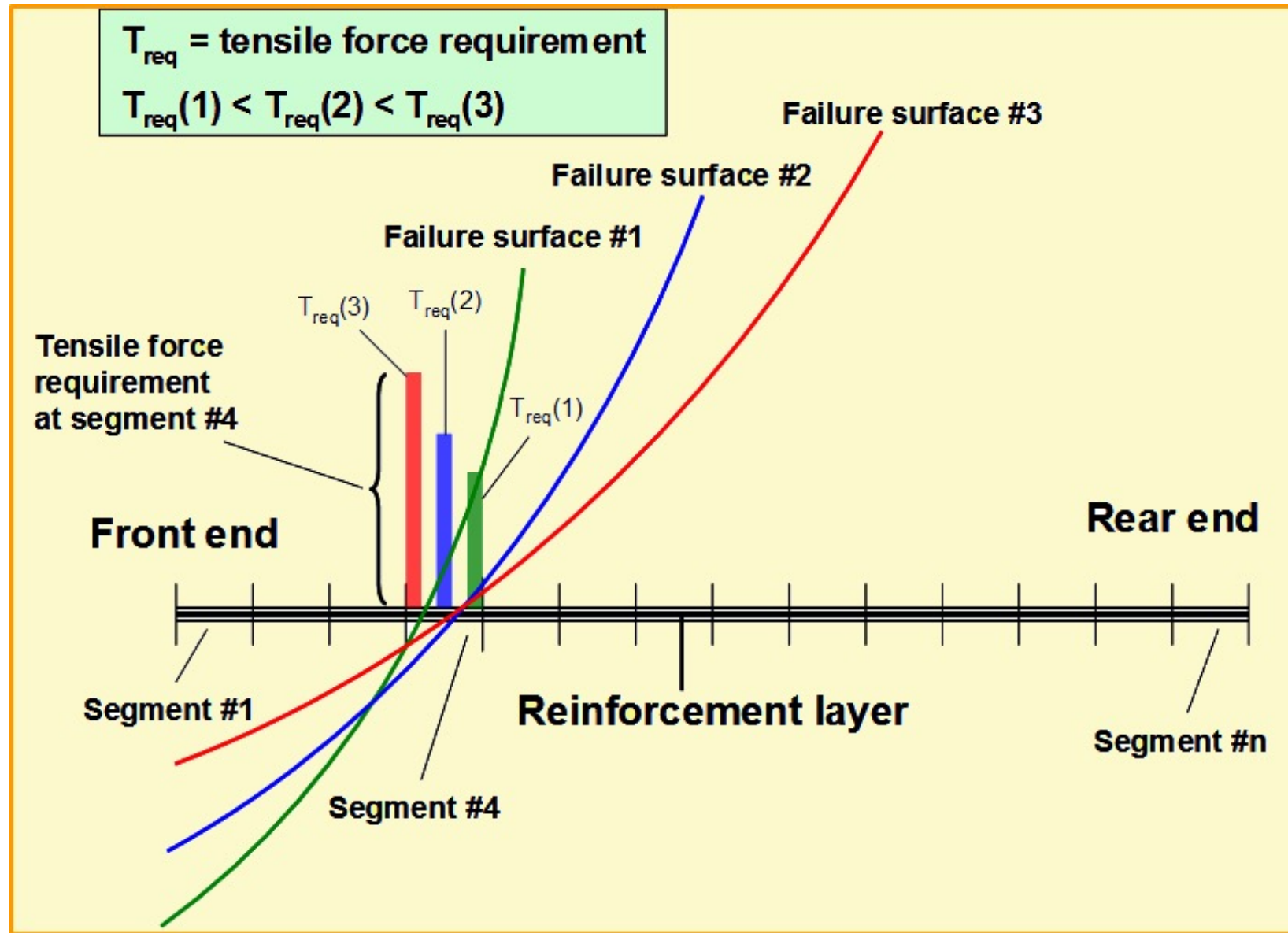
Tension Map: Visualization of $T_{req}(x)$



The Framework: **Process in Nutshell**

- Check numerous test bodies setting $SF=1.0$ and calculating $T_{req}(x)$ for each layer → Use a systematic top-down process
- For $T_{req}(x)$ distribution, failure along any surface is equally likely → $T_{req}(x)$ therefore is termed **Baseline Solution** → **Tension Map**
- The tension, $T_{req}(x)$, is limited by pullout at the rear and/or front ends
- $T_{req}(x)$ is the resistance needed locally to yield a structure at a limiting equilibrium state

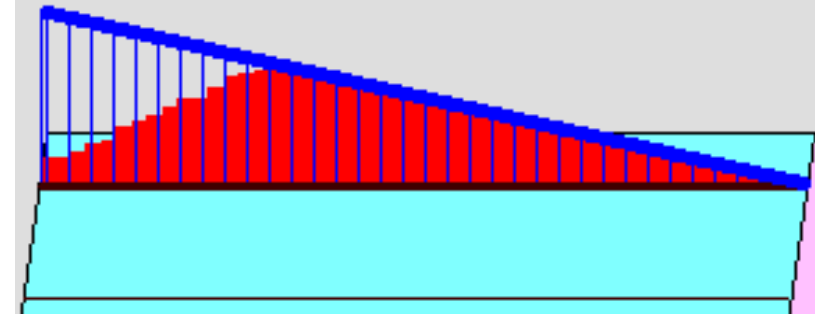
Maximization Update...



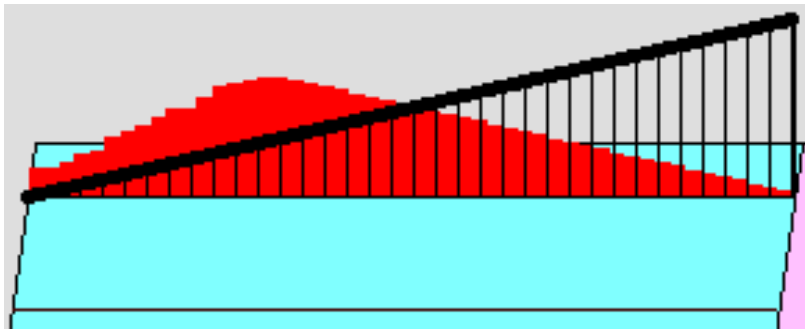
Details: **Baseline & Pullout**



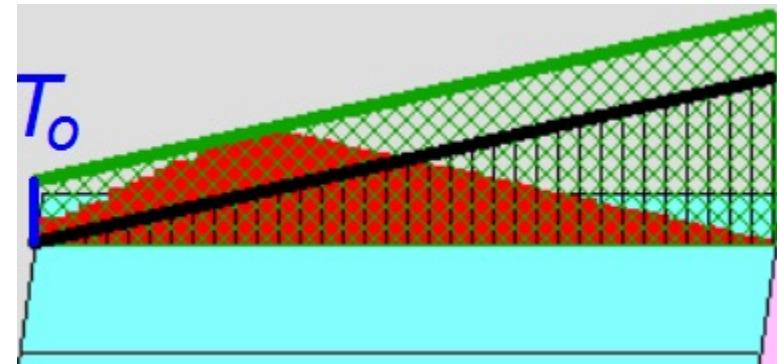
1. $T_{req}(x)$



2. *Rear pullout constraint*



3. *Front pullout... oops*



4. *Adjust front pullout*
→ *Upwards shift is T_0*

Geosynthetic Reinforced Wall (Alabama, Photo: Feb 2007)



Percolating water → Decrease in σ_v' → Decrease in front pullout resistance → Geogrid may not mobilize the needed resistance thus relying on added resistance from connections → Connections strength for upper layers exceeded → Failure

Can We Use ‘Better’ LE Methods for Baseline Solution?

- Yes, we can...
- Recall the term *framework* presented here— it is *not* restricted to a specific method of analysis
- Han and Leshchinsky (2006) used Culmann - instructive but has limited use
- Leshchinsky et al. (2014) used log spiral - rigorous but not easy to use (also, limited to homogenous problems)
- Leshchinsky et al. (2017) used Bishop - not rigorous but practical
- YOU may use rigorous a LE (e.g., Spencer, M-P) with general slip surface. You can even use **Limit Analysis of Plasticity**...

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Advancement of Current Design

- Apply the LE design approach in two stages:
Internal Stability and *Global Stability*
- **Stage 1:** Internal stability - Find $T_{req}(x)$ in all reinforcements - Baseline Solution
- Consider geometry, loading conditions, reinforcement layout, pullout resistance, any batter, water, seismicity, etc.
- **Stage 2:** Global stability - consistent with current design → Standard slope stability analysis

Stage 1: Internal Stability

- Find $T_{req}(x)$ including T_{max} & T_o (connection)
- Determine $\max(T_{max})$ to select geosynthetic
- $LTDS = F_{s-strength} \times \max(T_{max})$ where $F_{s-strength} = 1.5$
- $T_{ult} = LTDS \times RF_{cr} \times RF_d \times RF_{id}$
- **Stage 1** is a rational and robust alternative to existing approaches → Ensures that there is no overstressing of reinforcement

Stage 2: Global Stability

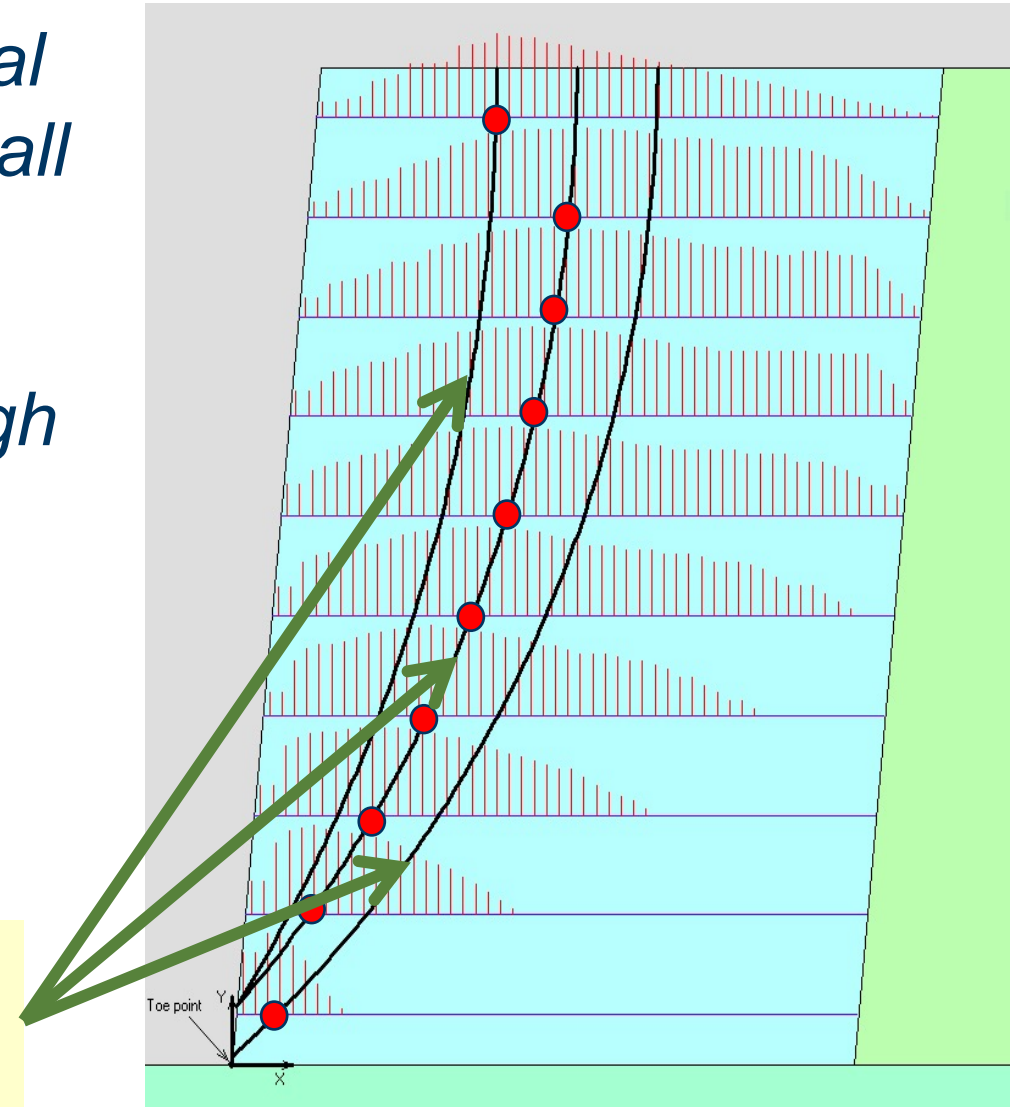
- Select reinforcement and facing following Stage 1
- Conduct global slope stability analysis to ascertain that for the selected facing, layout and strength reinforcement, $F_s \geq 1.30$ for all feasible failure geometries
- Increase the length and/or strength of reinforcement, if needed to meet the prescribed on soil strength F_s

Stage 2 Conducts Global Stability

Why use then Internal Stability?

- Reinforcement resistance in Global Stability is evenly divided amongst all layers \rightarrow Results in T_{max} that is smaller than in Internal Stability \rightarrow Global ignores local demand through 'smearing'
- Global Stability tells us nothing about connection load, T_o

Global Stability: Locus of T_{max} is NOT on a singular surface.



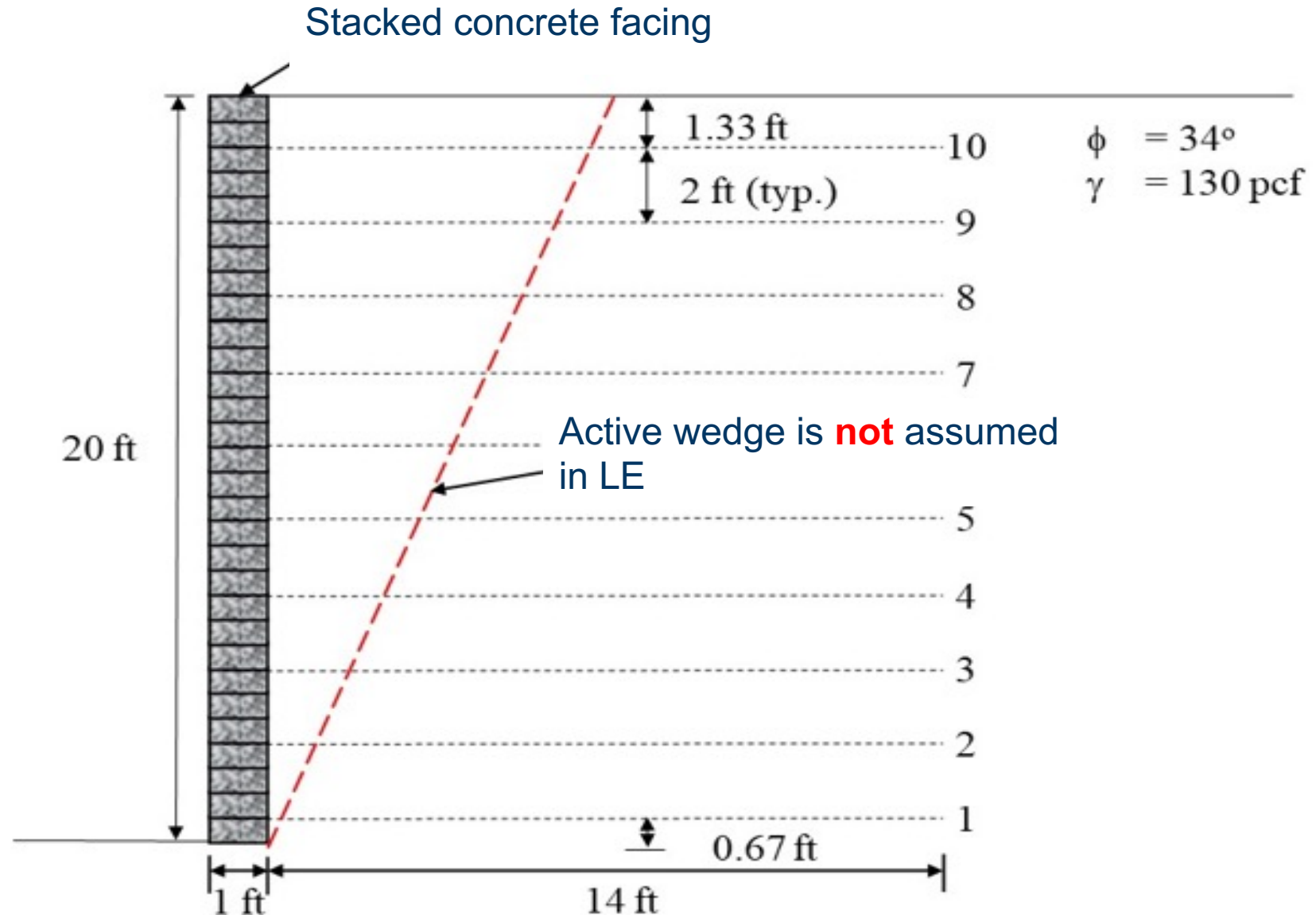
So Why Stage 2 is Important too?



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Example Problem



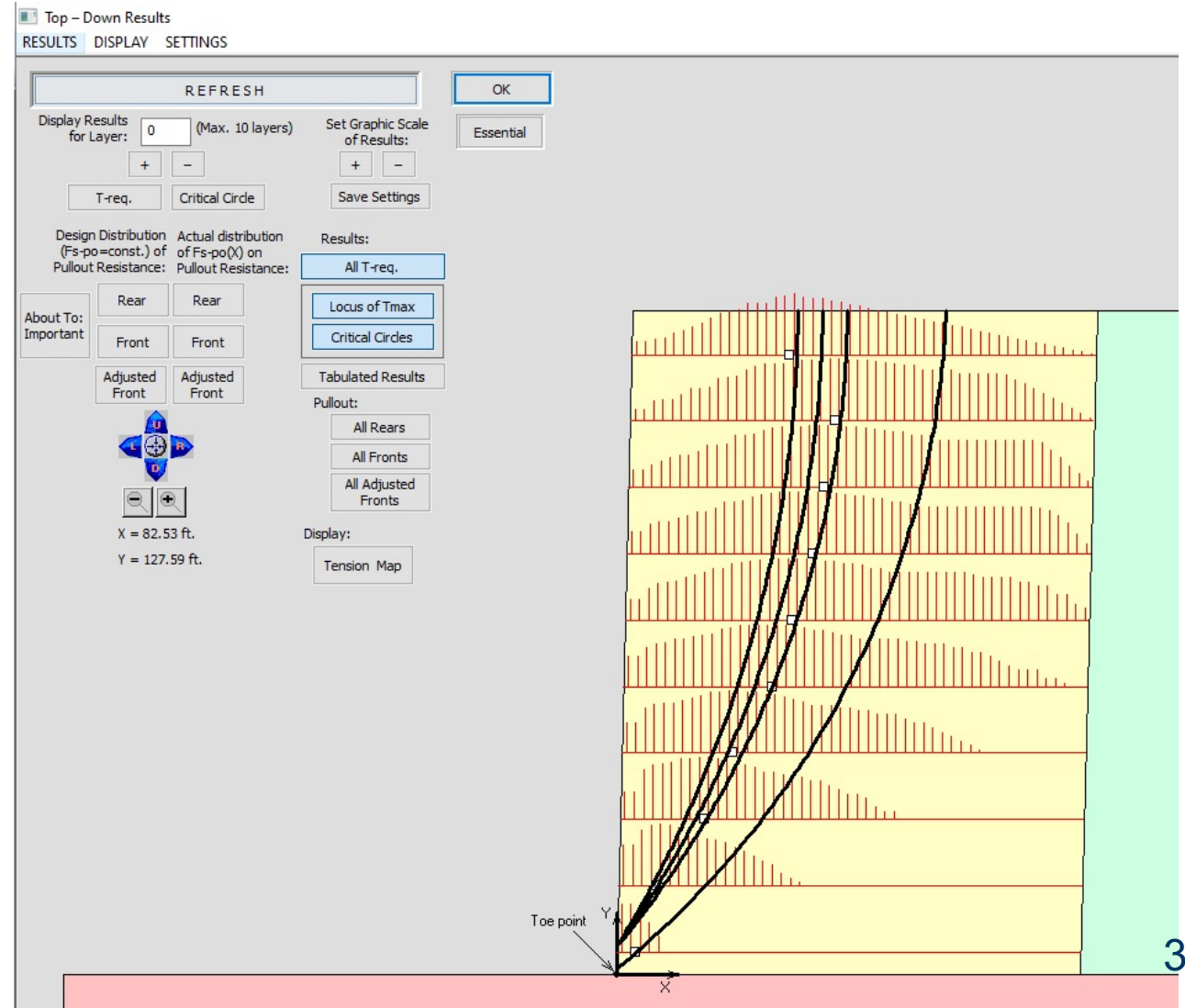
Stage I: Get $T_{req}(x)$ and T_{max}

Objective:

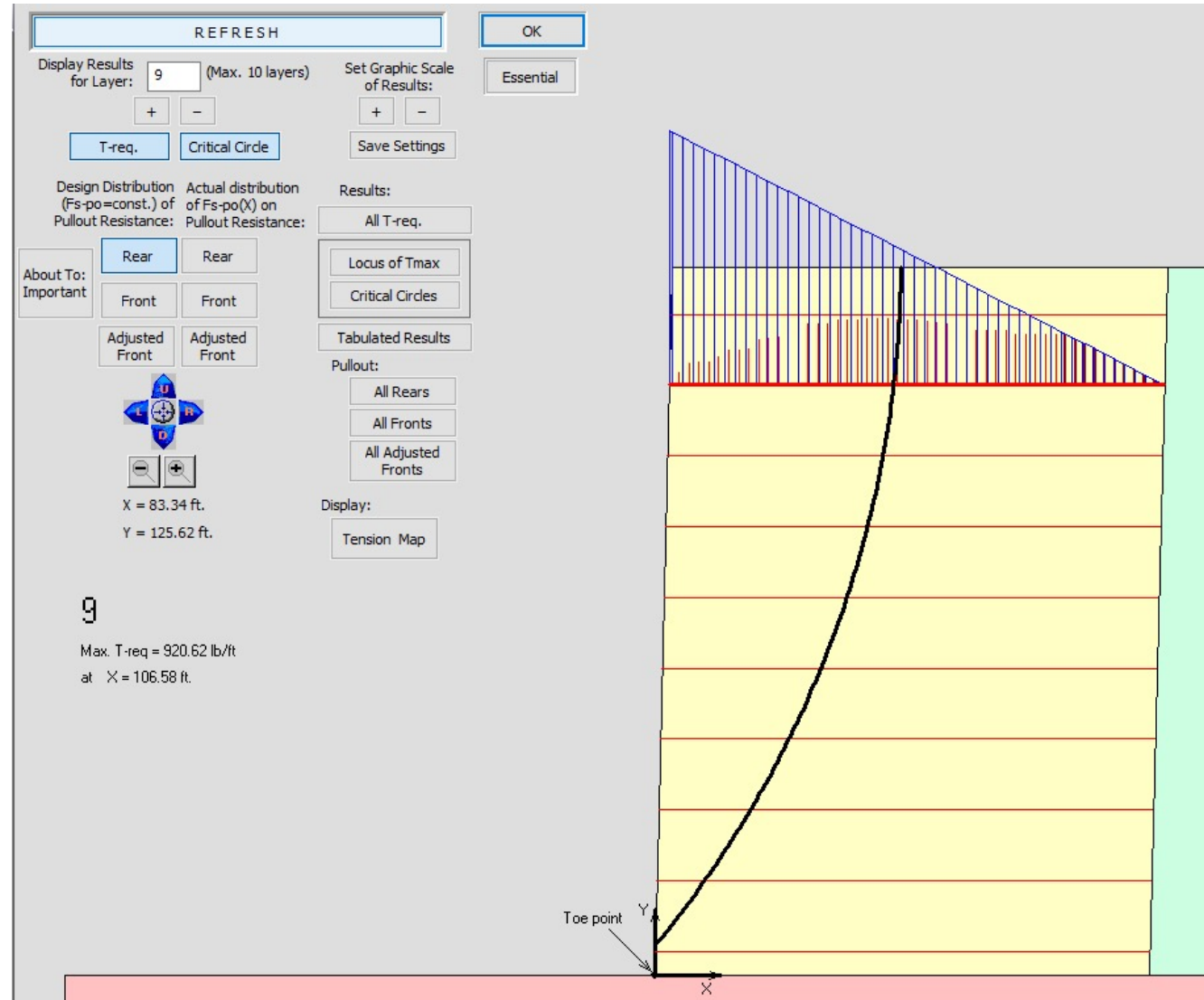
Find $T_{max} \rightarrow T_{ult} = 1.5 \text{ LTDS}$
 $= 1.5 RF_{id} RF_d RF_{cr} T_{max}$

If reinforcement strength is same as $T_{req}(x)$, any circle through layers will have the same $F_s = 1.0 \rightarrow$ All circles are equally critical \rightarrow Baseline results are rendered to select reinforcement with adequate T_{ult} ensuring sufficient margins of safety

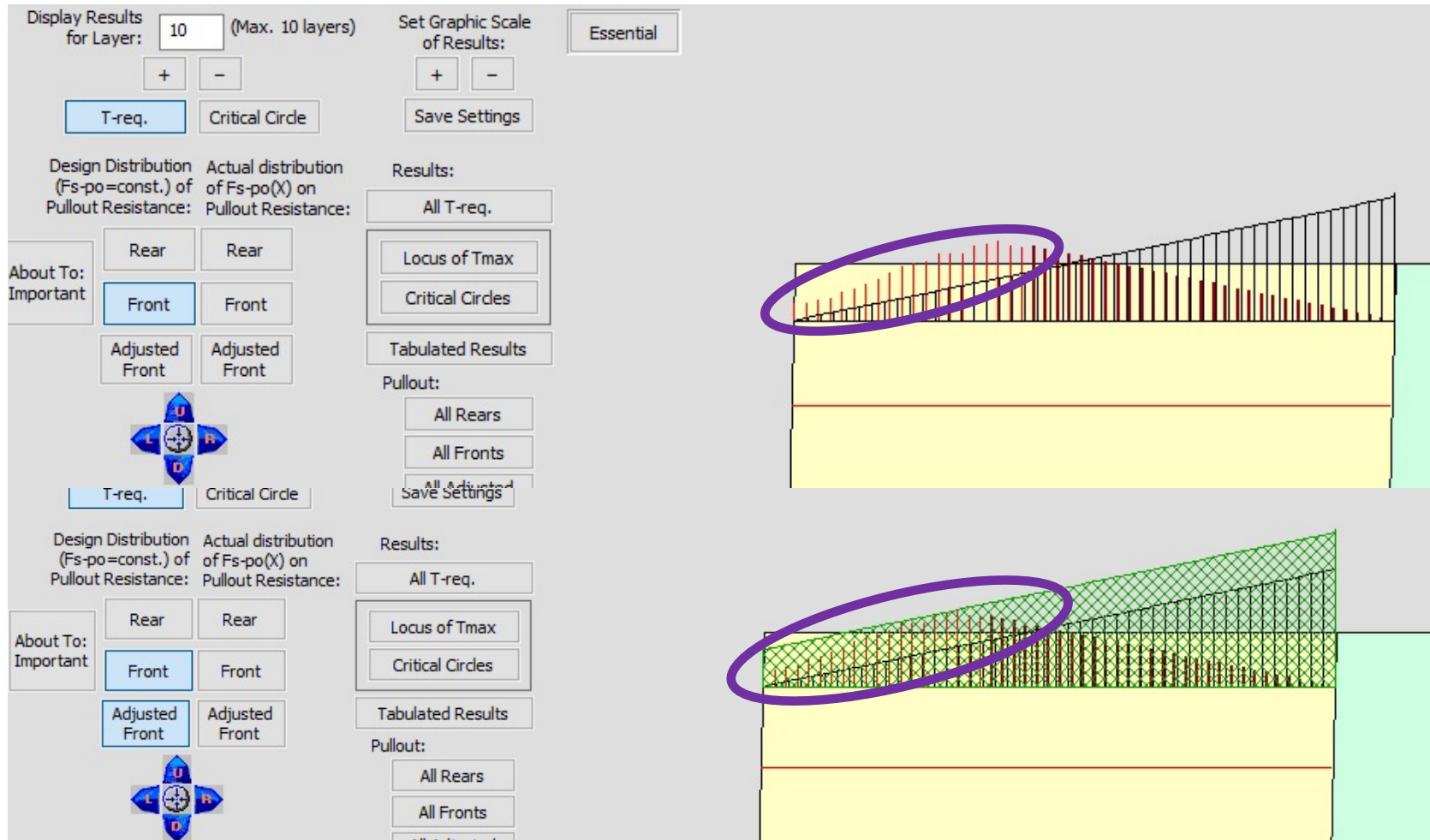
Note: There is NO well-defined Active Wedge as postulated in most simplified designs



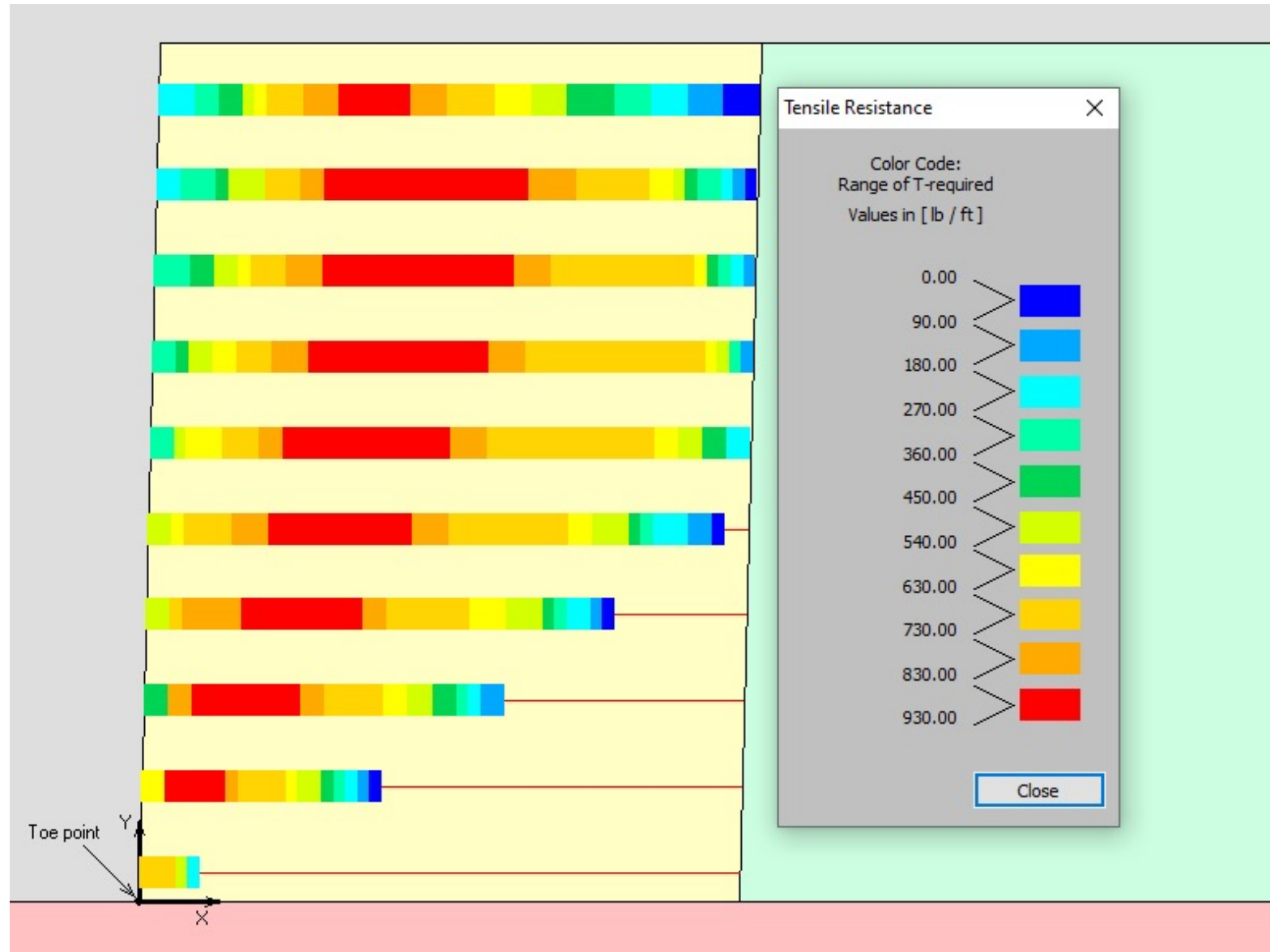
Circle for Layer 9 Determining T_{max} . Note that T_{req} is limited by Pullout Resistance thus Shedding load to layers below



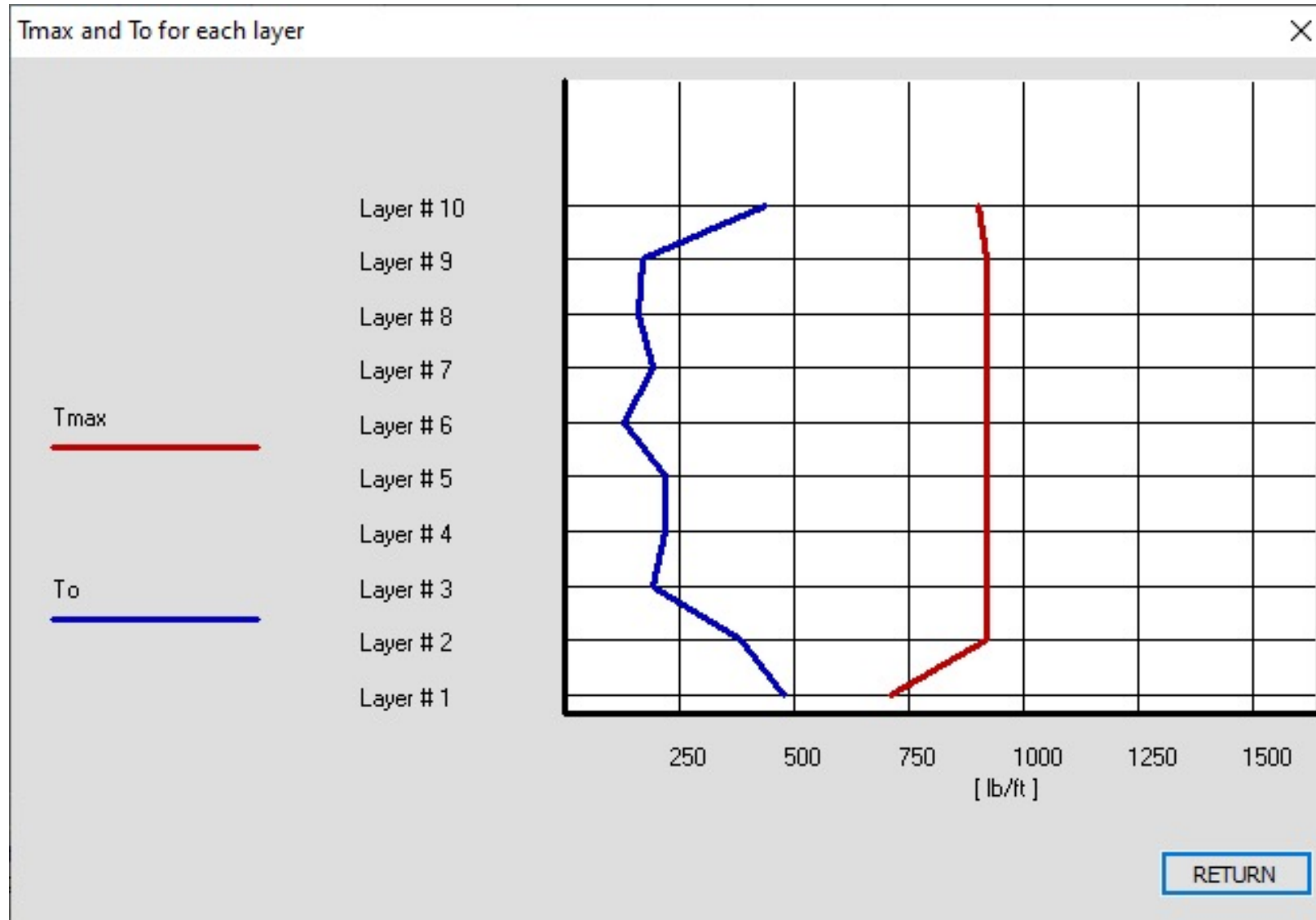
Determining To: For Treq, frontend pullout resistance must be satisfied



Tension Map: Color Coded Visual of T_{req}



T_{max} and T_o Distributions



Estimate Horizontal Displacement

Estimated Horizontal Displacement, d, at Face of Slope for Specified Fs

Layer No.	Height from Toe	Current input of LTDS	Tensile Modulus of Geosynthetics, J, at 2% strain	Horizontal Displacement at Face of Slope, d
	[ft]	[lb/ft]	[lb/ft]	[inch]
1	0.67	920.79	34000	0.28
2	2.67	920.79	34000	1.17
3	4.67	920.79	34000	1.88
4	6.67	920.79	34000	2.49
5	8.67	920.79	34000	3.05
6	10.67	920.79	34000	3.38
7	12.67	920.79	34000	3.48
8	14.67	920.79	34000	3.38
9	16.67	920.79	34000	3.20
10	18.67	920.79	34000	2.44

CALCULATE

Layer # 10

Layer # 9

Layer # 8

Layer # 7

Layer # 6

Layer # 5

Layer # 4

Layer # 3

Layer # 2

Layer # 1

5

[inch]

NOTES:

1. The approximated horizontal displacement at the face of the slope is appropriate for limit state; i.e., when your specified $F_s=1.0$ in top-down approach leading to full mobilization of the soil strength considering rotational slip surfaces.

2. The approximated horizontal displacement, d, is calculated following this expression:

$$d = \sum_{i=1}^n \left(\frac{\tau_i}{J} \right) \Delta X_i$$

Where: τ_i is the force calculated at segment i and

ΔX_i is the length of segment i.

That is, $\Delta X_i = L / n$ where L is length of the considered reinforcement layer and n is the number of segments along a layer specified in your data (between 50 and 200). J is the tensile modulus of the reinforcement having unit of [Force/Length]. Typically, J is determined at 2% geosynthetic strain.

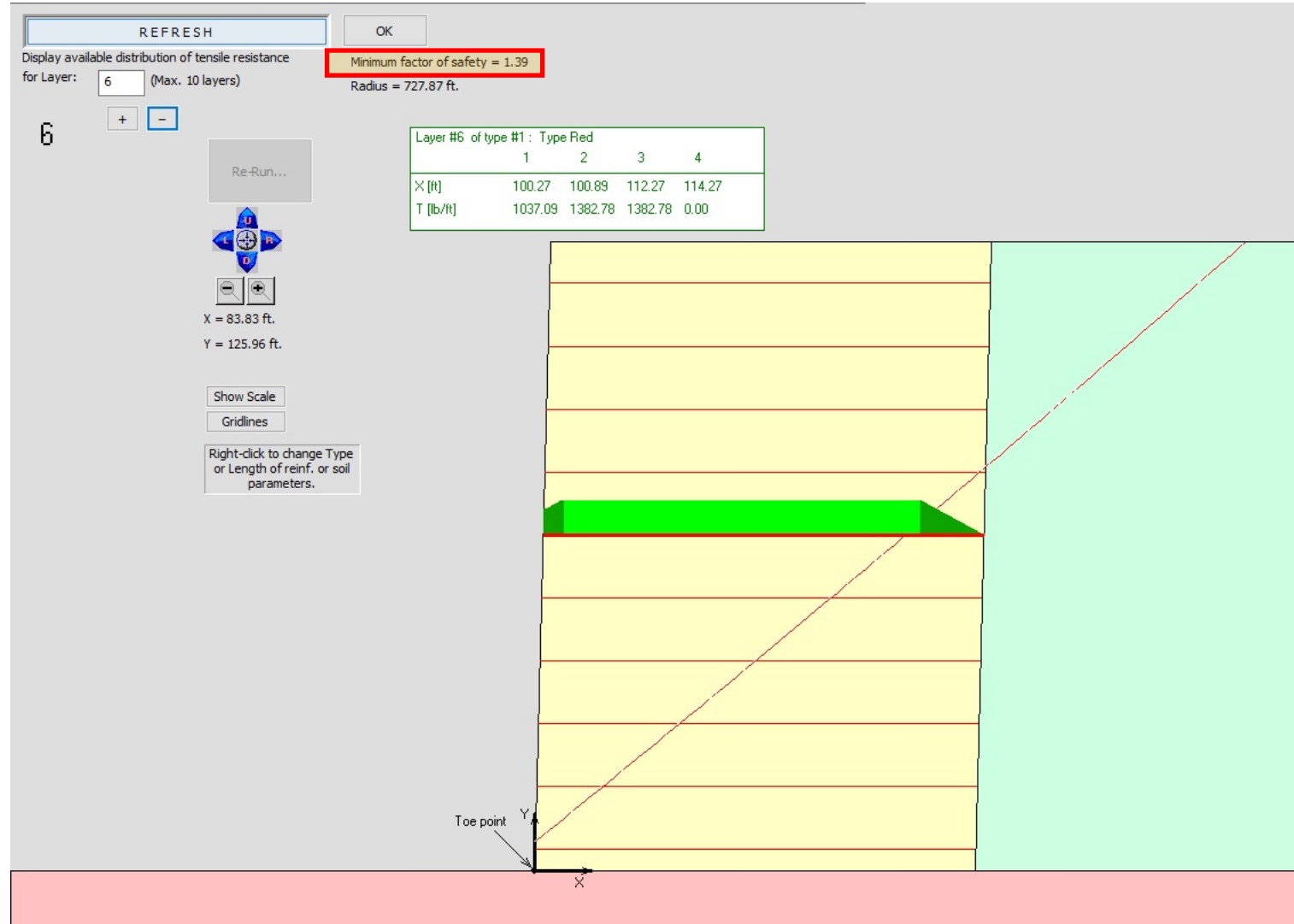
3. The displacement d is solely due to estimated cumulative elongation of the reinforcement. It does not reflect possible translational movement of the reinforced mass. To avoid translational movement, conduct 2-part wedge global stability analysis (in Global Stability mode) verifying that for the selected layout of reinforcement the global F_s is adequate, typically >1.3 .

DEFAULT

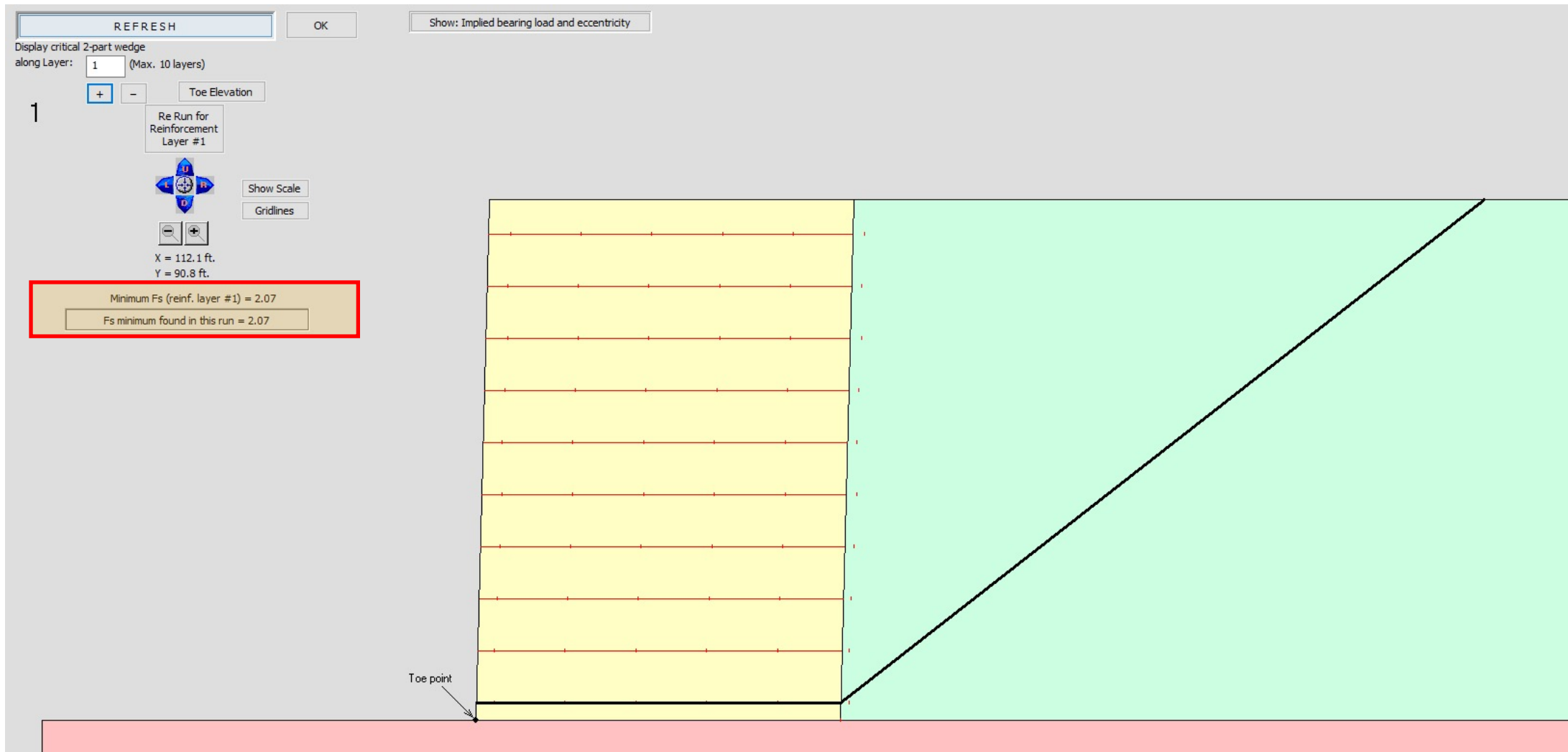
OK

Cancel

Stage II: $T_{ult} = 1.5 \text{ RF } T_{max} = 3020 \text{ lb/ft}$
Run Global Stab.: $Fs = 1.39 > 1.30 \text{ OK}$

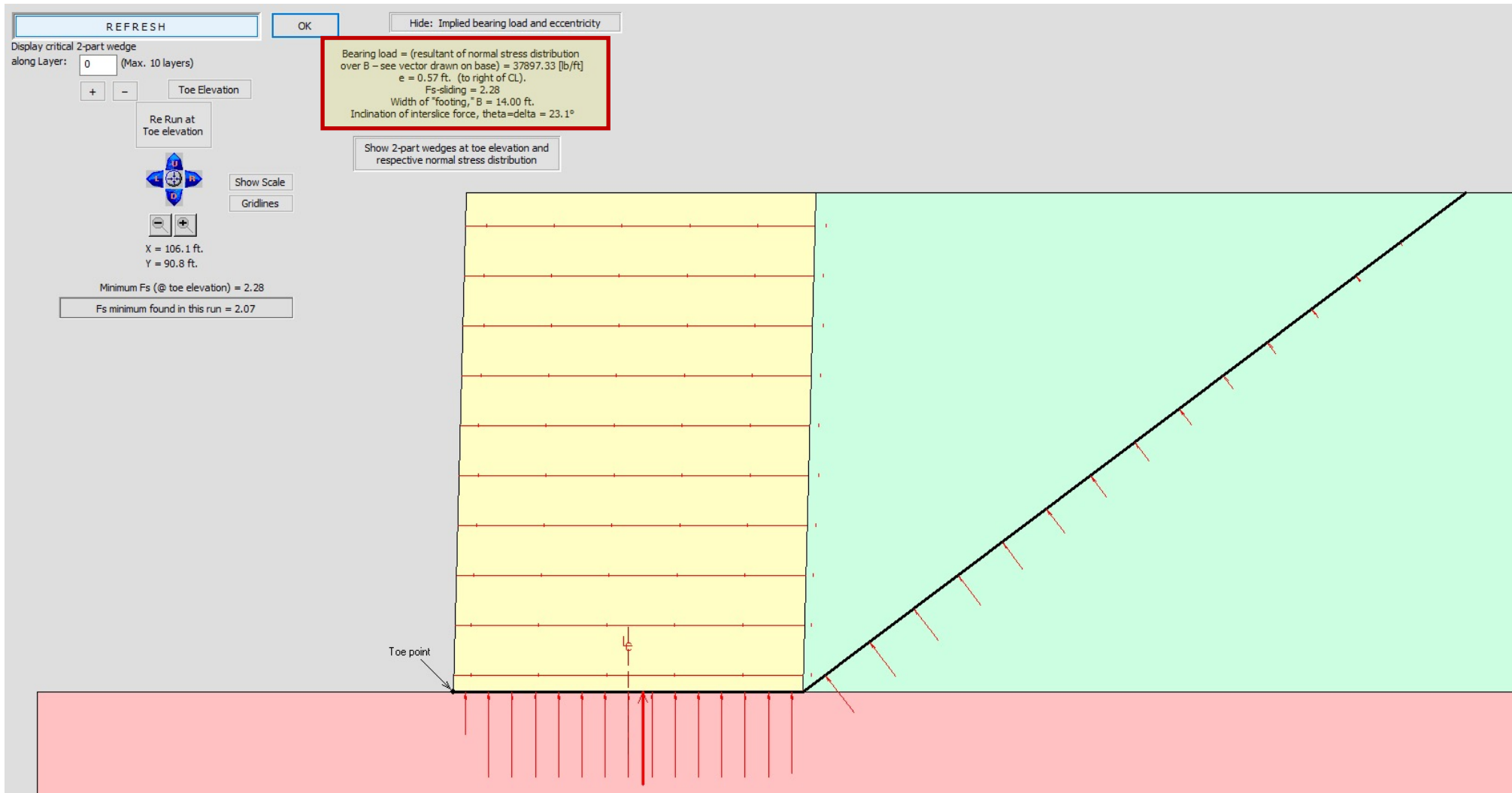


Run 2 Part Wedge Sliding using Spencer – $F_s=2.07$ OK

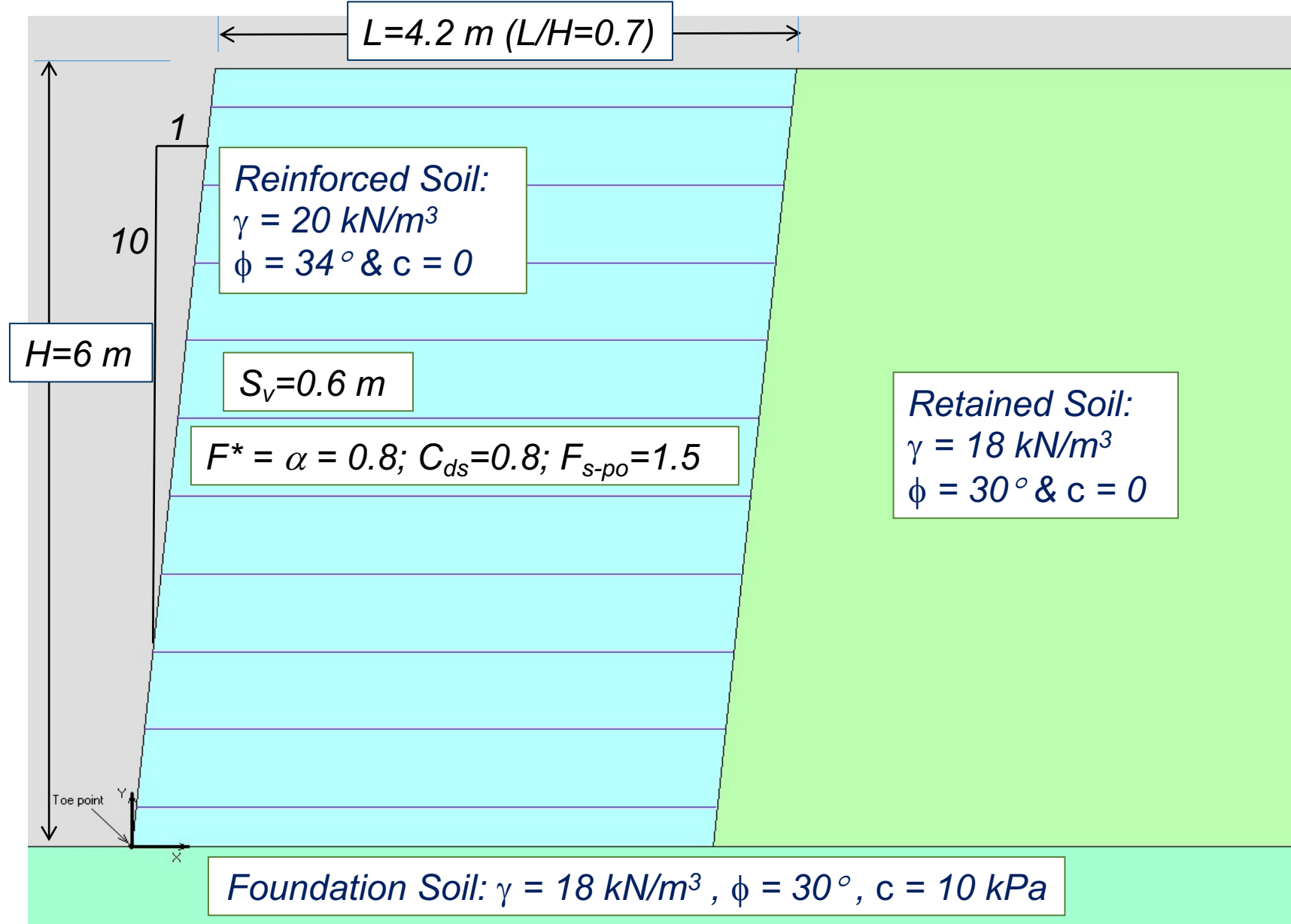


Using Spencer: Get Normal Stress

→ e, R and Meyerhof $\sigma_v = R/(L-2e)$



Benchmark Problem



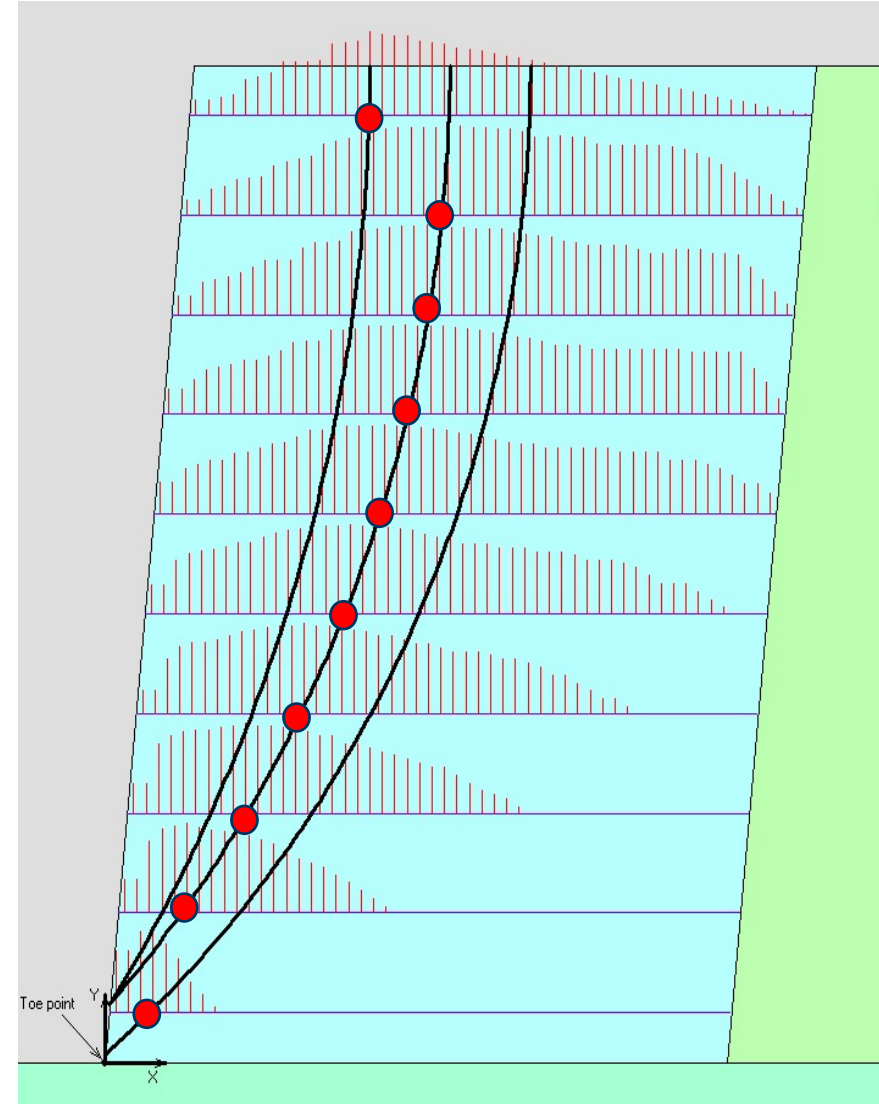
Computing T_{max} in Internal Stability: Critical Circles

1. Hypothesis in AASHTO: Locus of T_{max} is defined by a singular slip surface.

Is it?

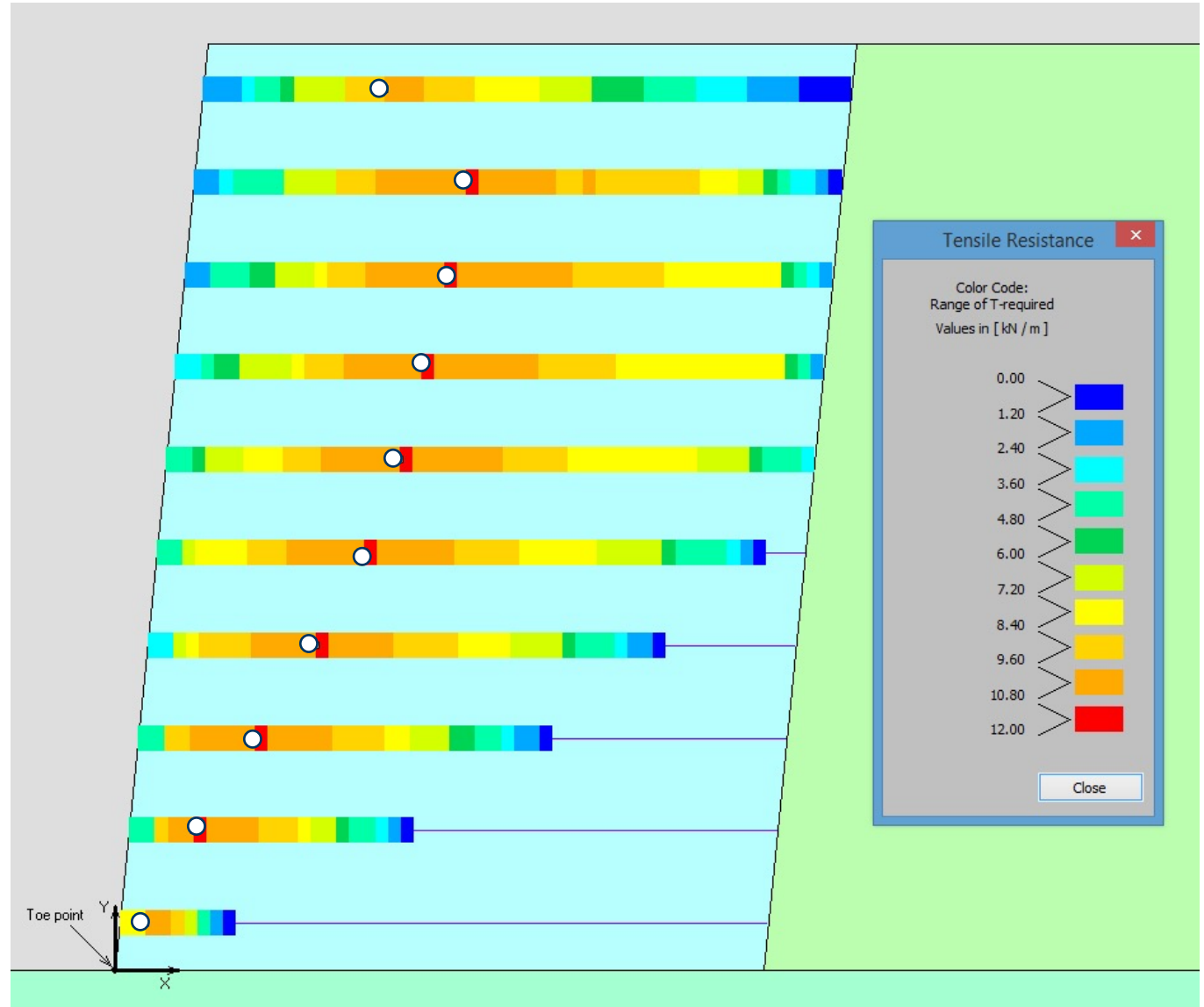
2. Well-defined active and resistant zones.

Is it?

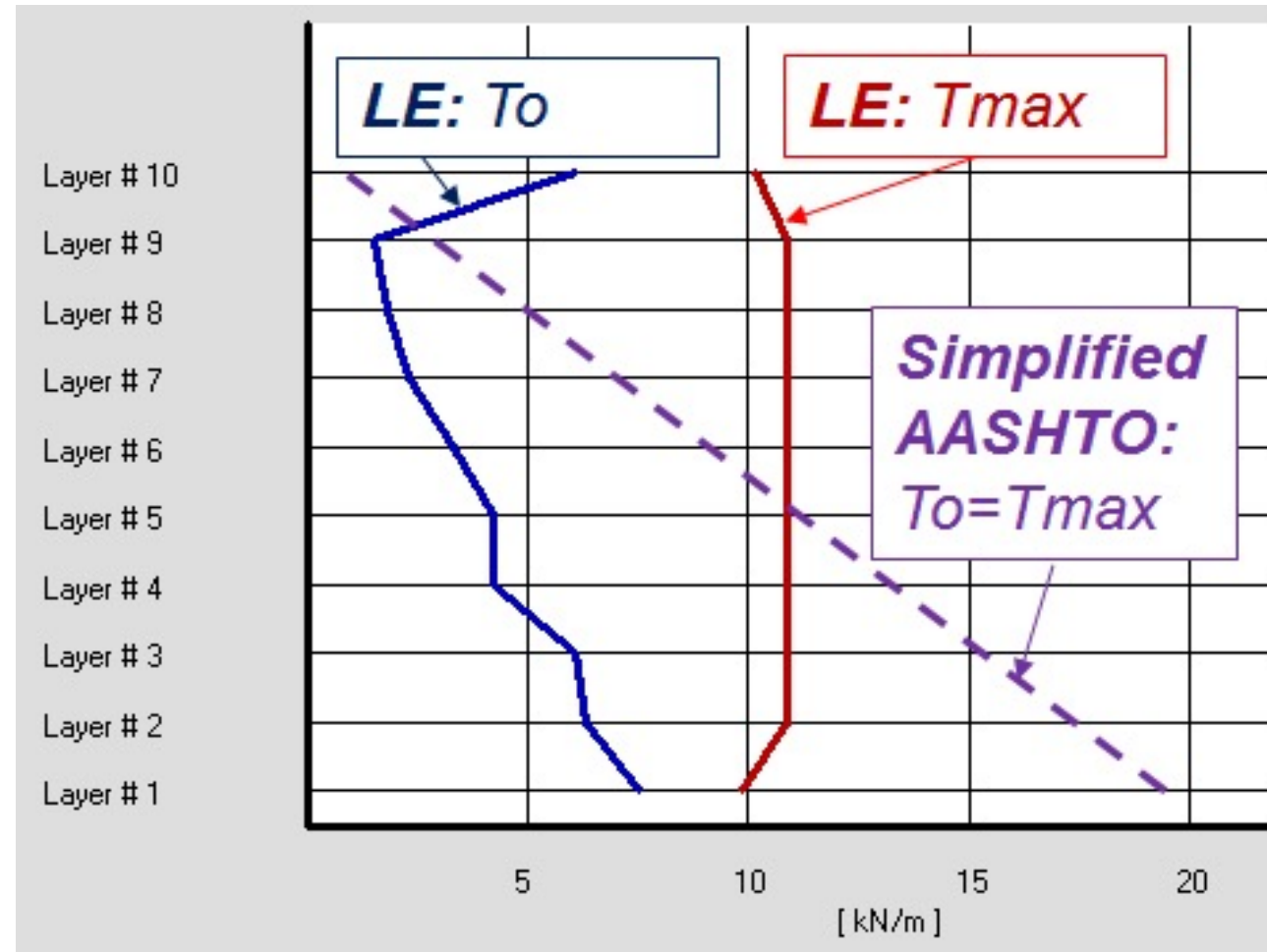


Tension Map

The mobilization of tension in each reinforcement can be visualized through the Tension Map →
Note location of T_{max}



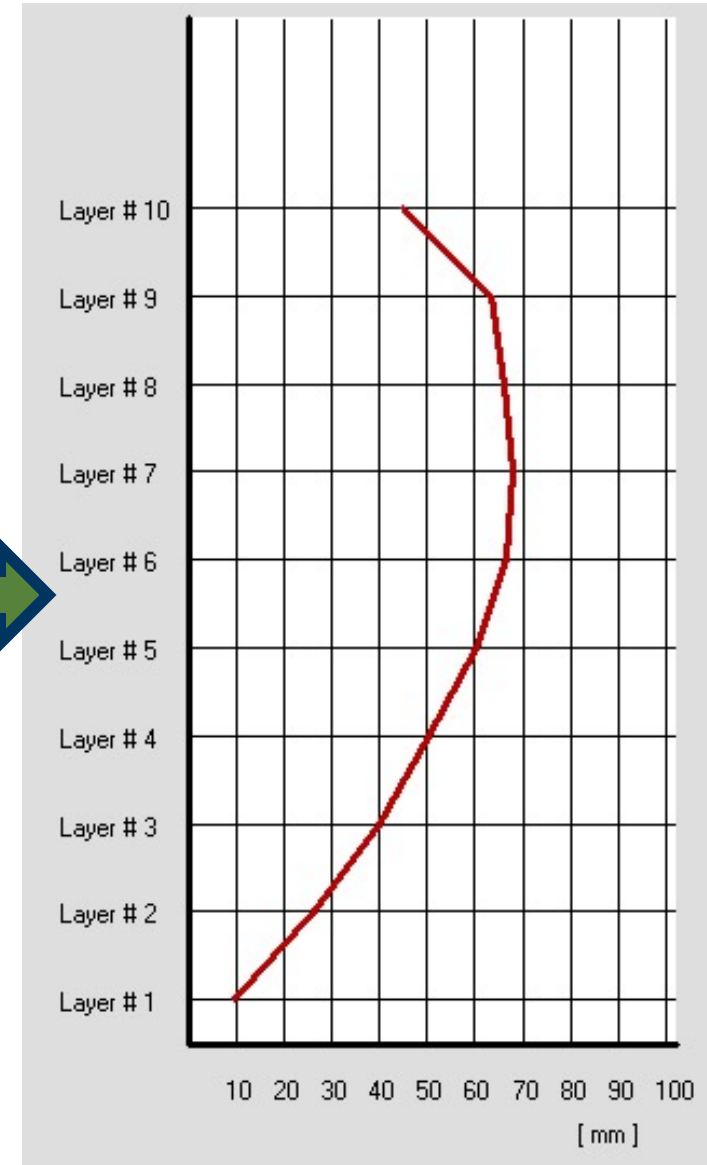
T_{\max} and T_o Distribution



$\max(T_{\max})$: **LE** \rightarrow 10.9 kN/m **AASHTO** \rightarrow 19.3 kN/m

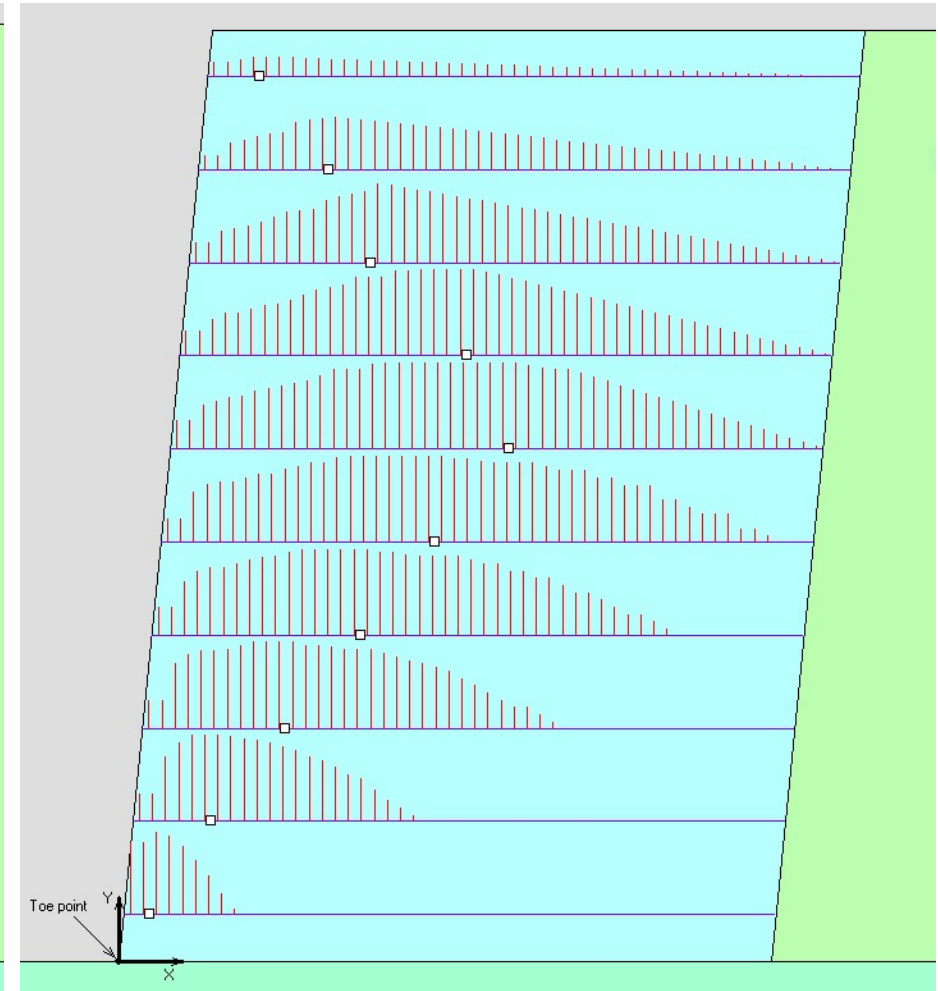
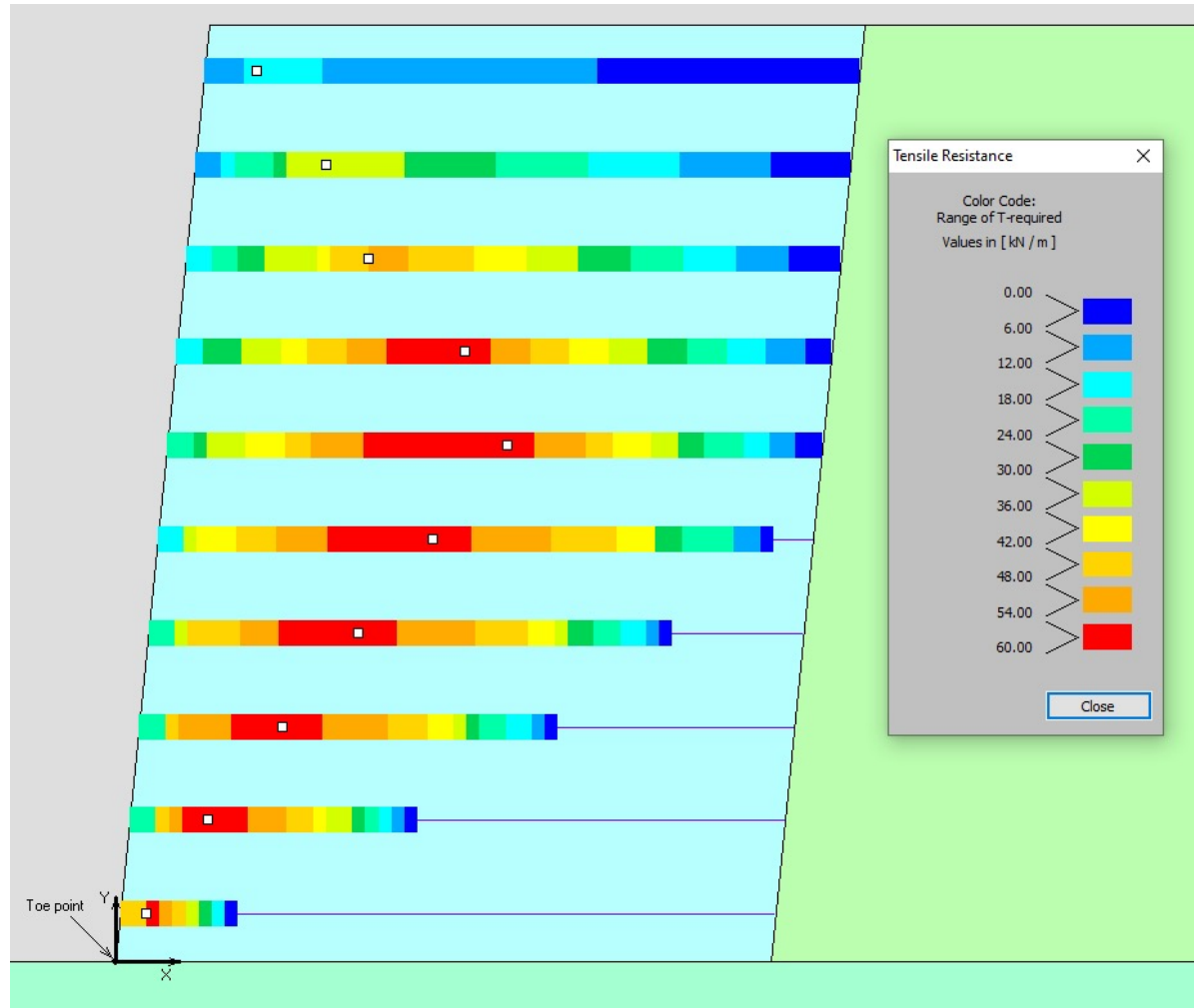
Horizontal Displacement Distribution

$T_{req}(x)$ for $F_s=1.0$ allows for **Estimation** of the lateral displacement at a limit state
e.g., for $J=500 \text{ kN/m}$

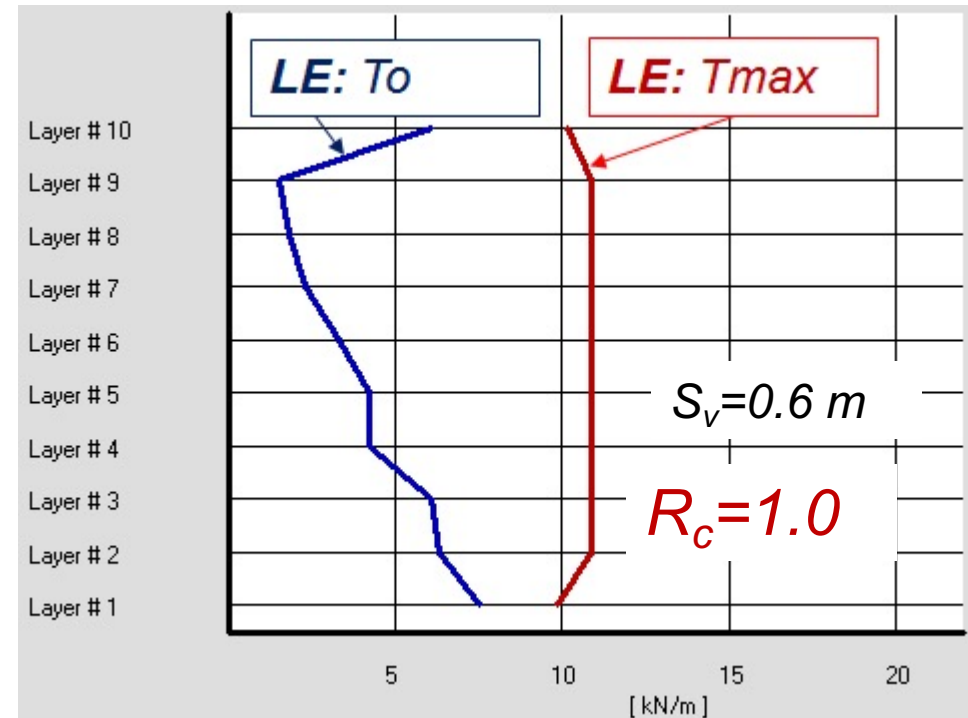
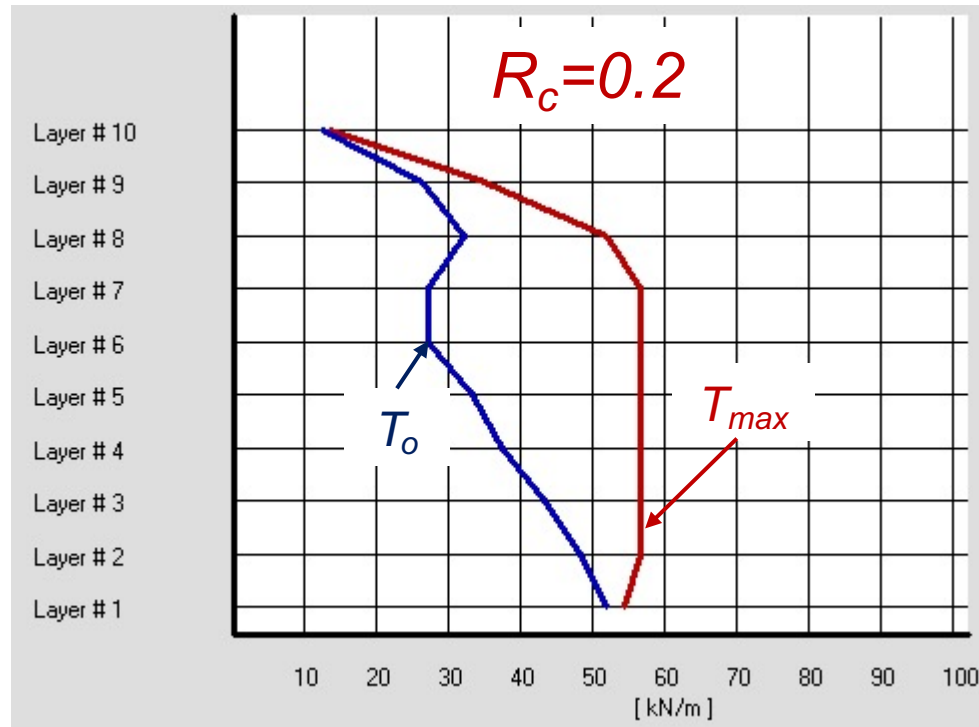


Computed Distribution of $T_{req}(x)$:

$R_c=0.2$

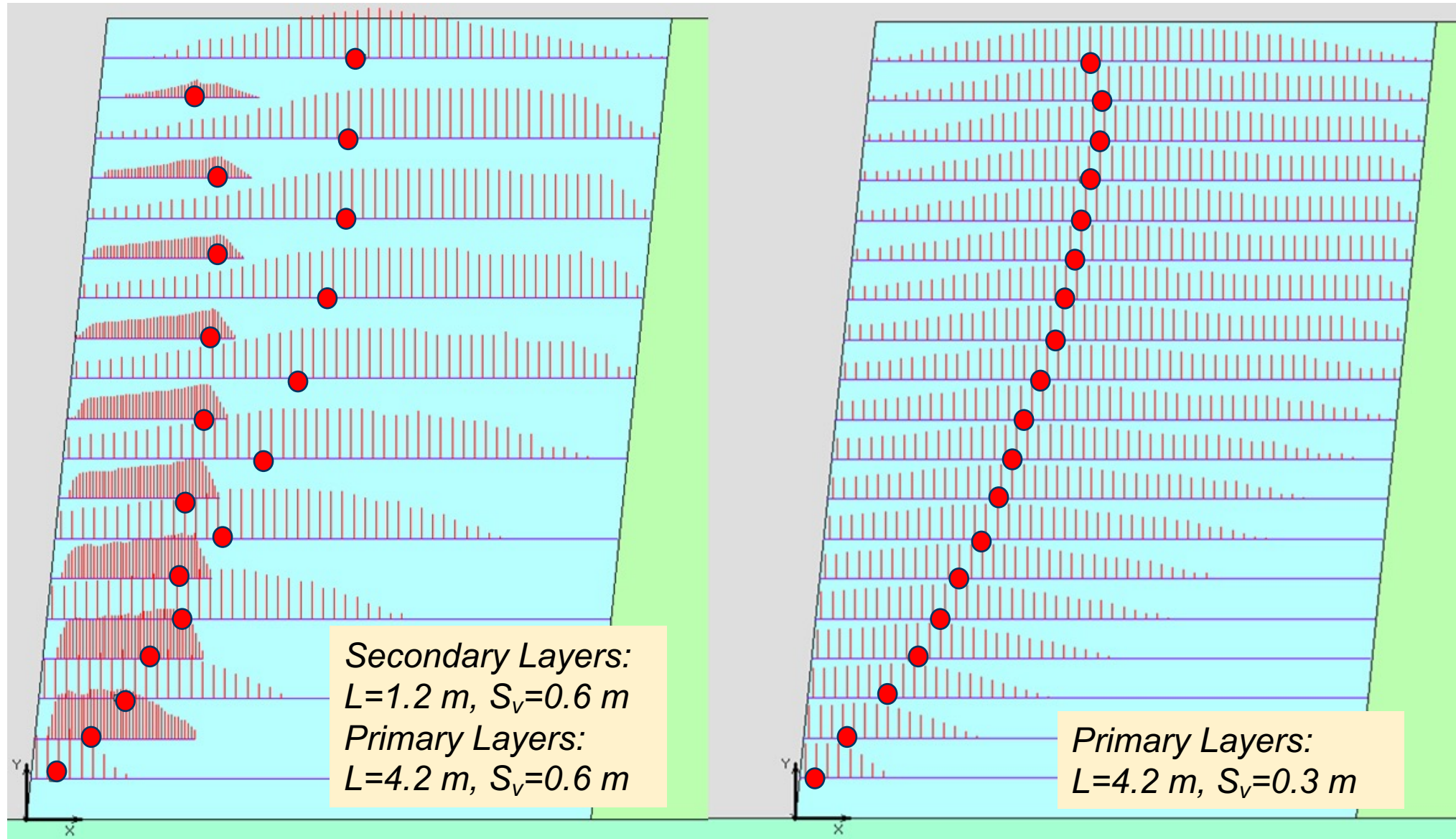


Effects of R_c on T_{max} and T_o

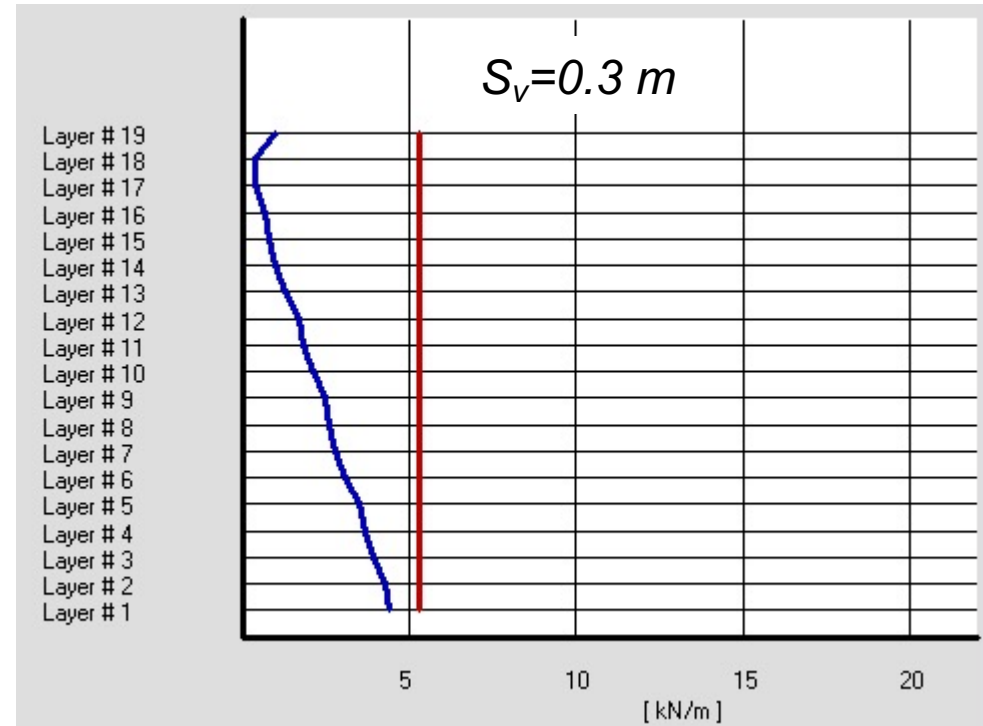
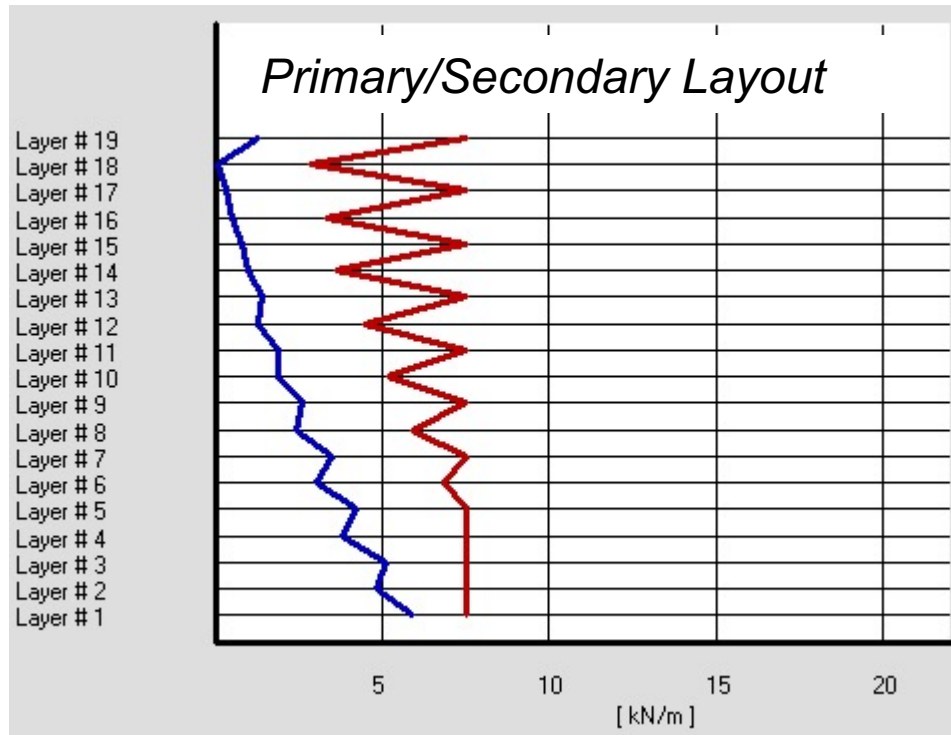


Note the different drawing scale for T_{max} and T_o

Effects of Secondary Layers

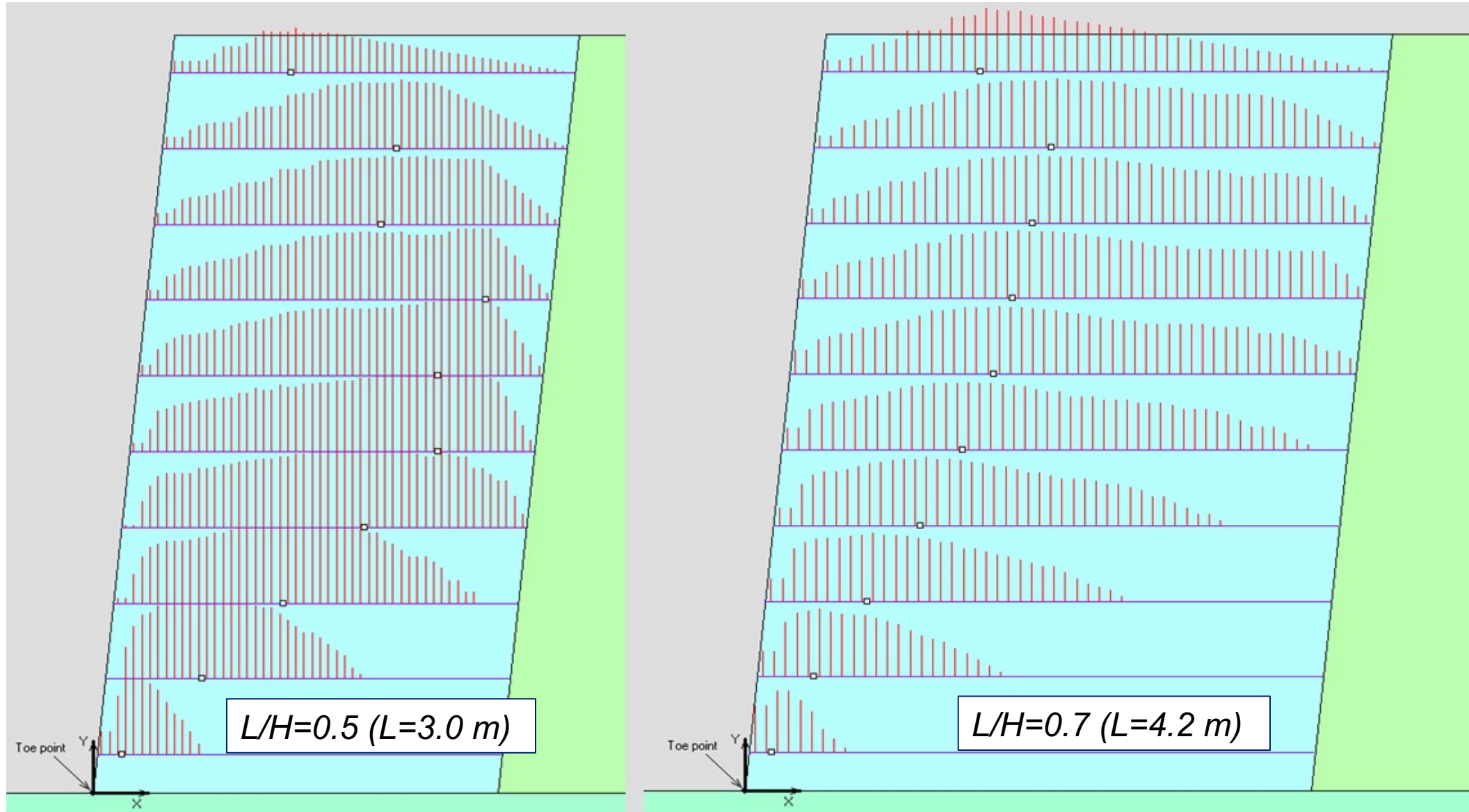


T_{max} and T_o : Secondary versus Close Spacing



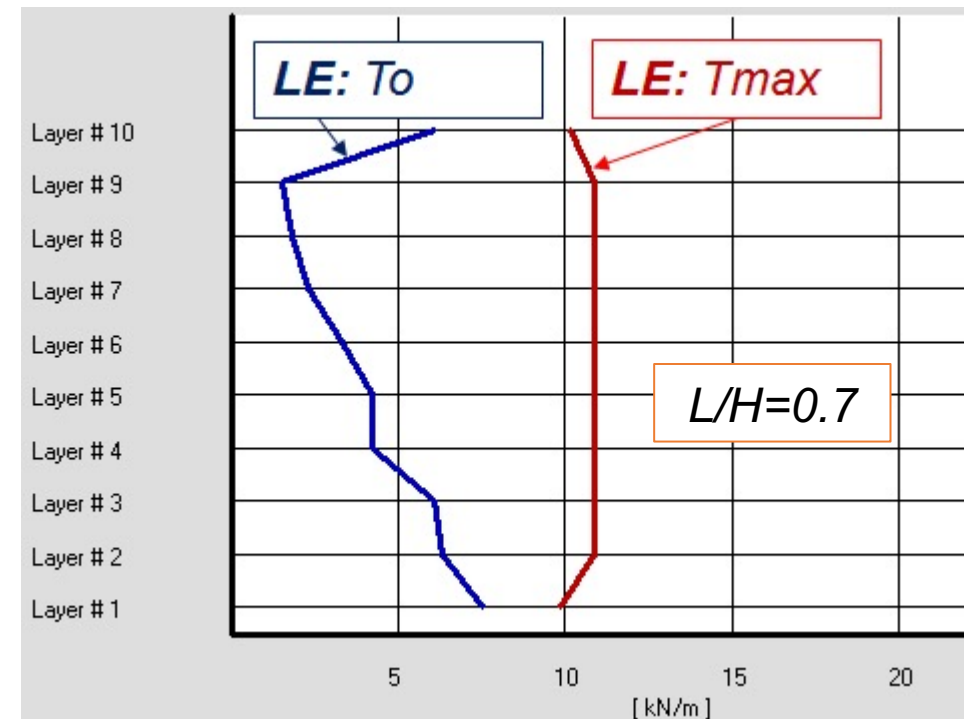
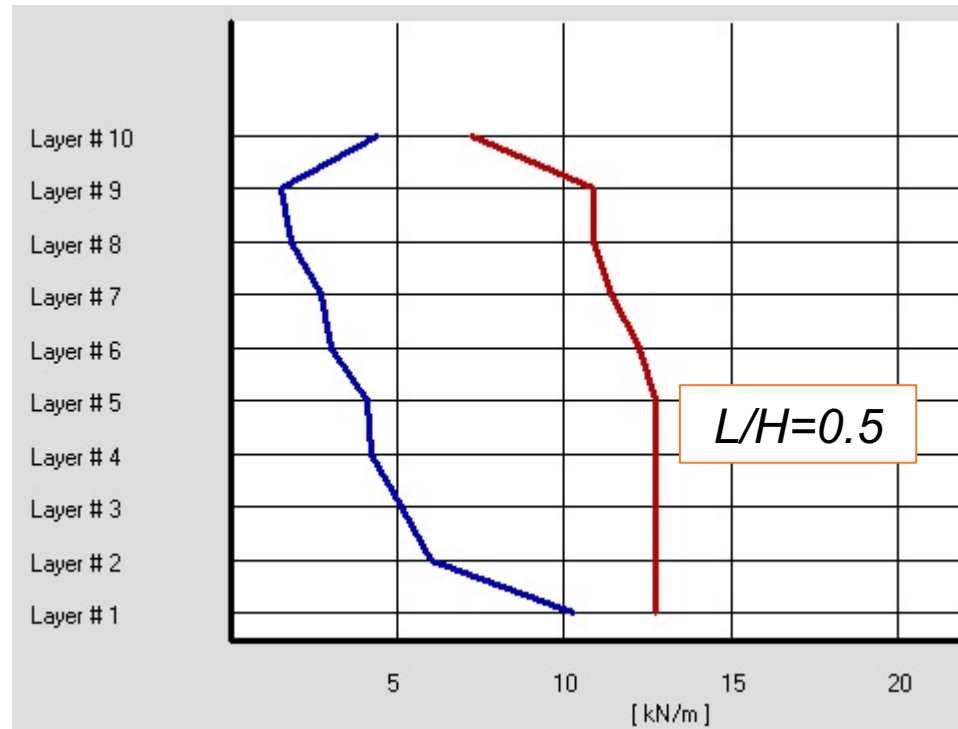
Depending on relative length of secondary reinforcement, it may decrease T_{max} . Generally it has significant effects on T_o (connection loads).

Effects of Shorter Reinforcement



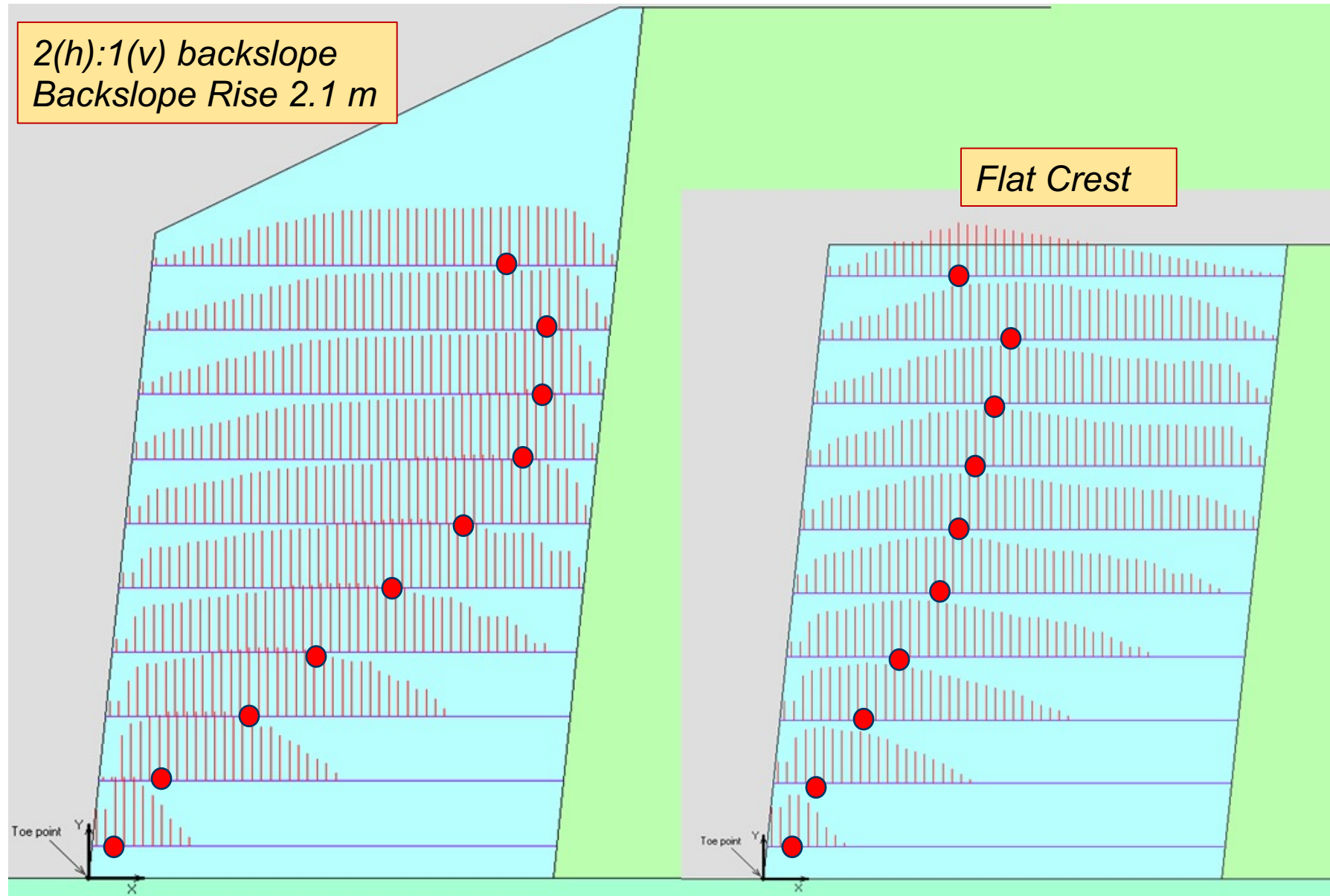
Effects of Shorter Reinforcement:

T_{max} and T_o



Generally, lower layers carry higher load due to compound failures →
Upper layers need to contribute less to produce $F_s=1.0$ → Top layer
carries less load thus resulting in smaller T_{max} and T_o

Effects of Backslope

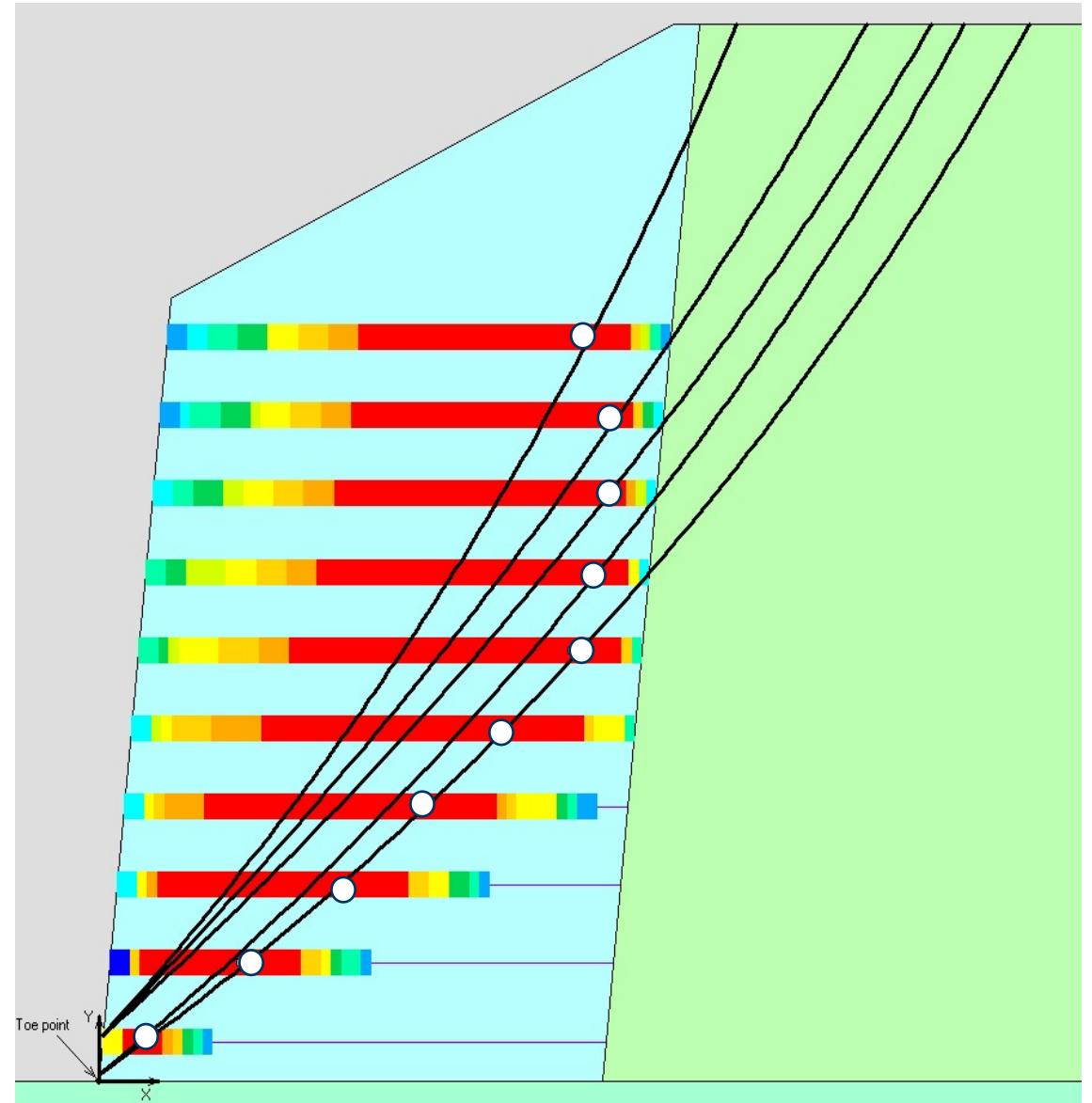


Computing T_{\max} in Internal Stability: Critical Circles

Note:

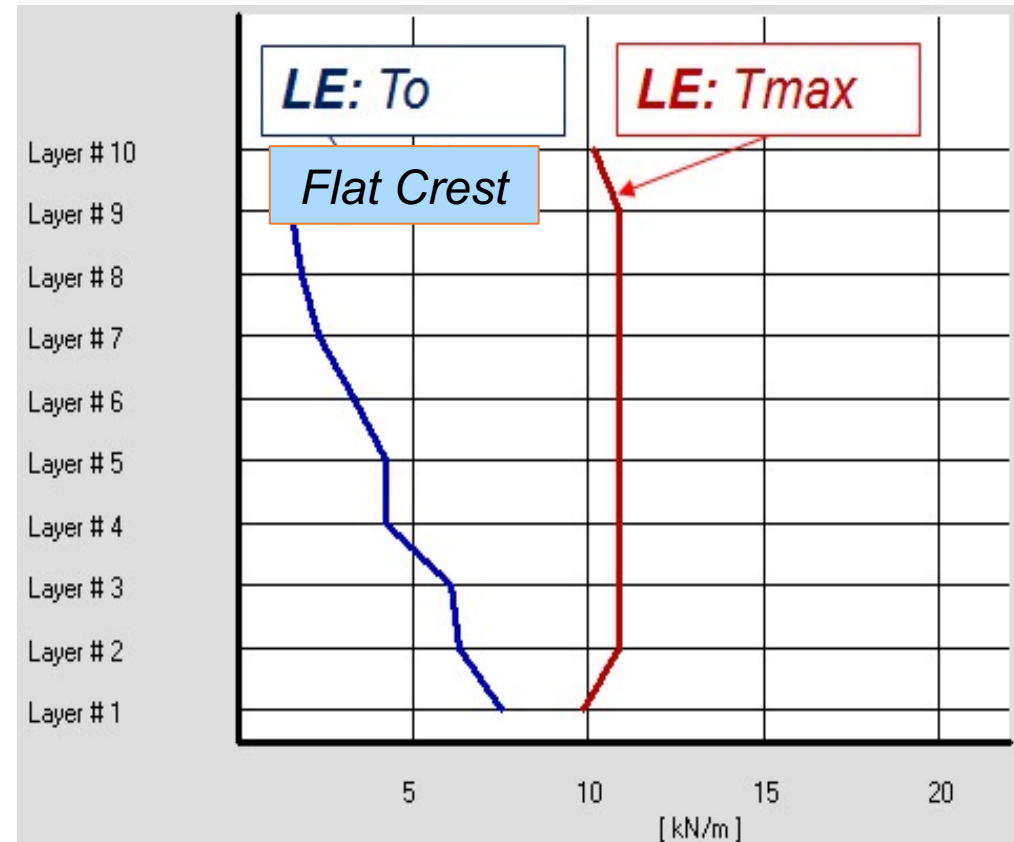
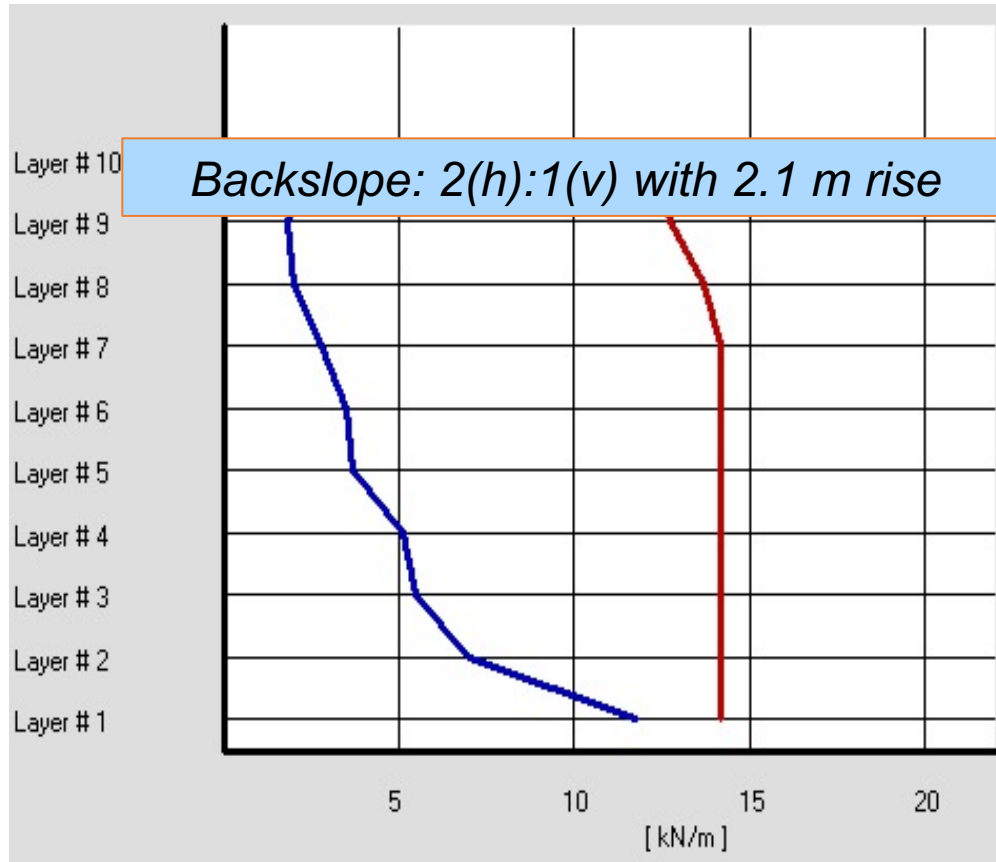
*Global Stability →
Top 4 layers are
not needed for
stability.*

*Baseline Solution,
Stage 1 →
Identifies the need
for these layers!*

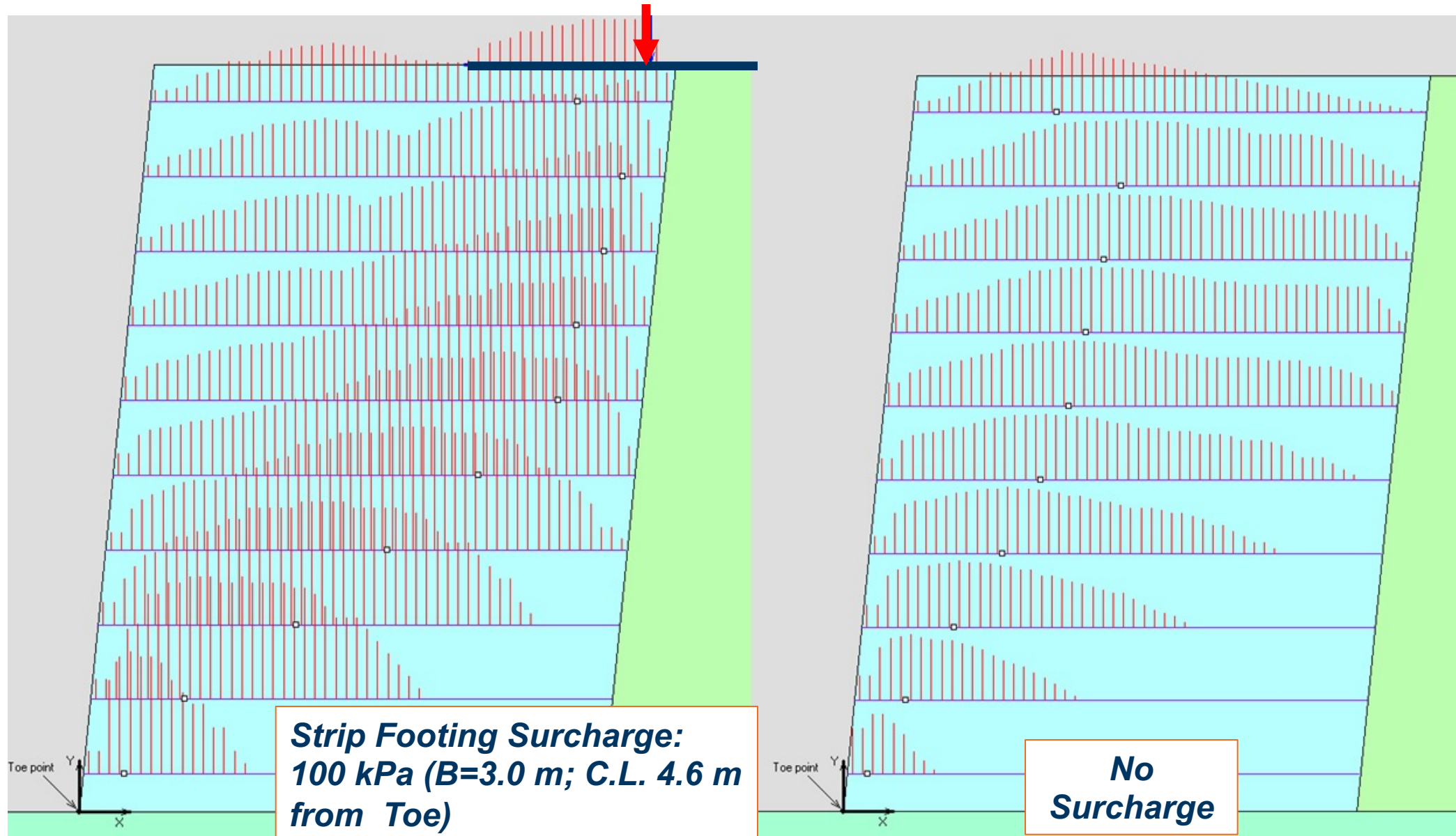


Effects of Backslope:

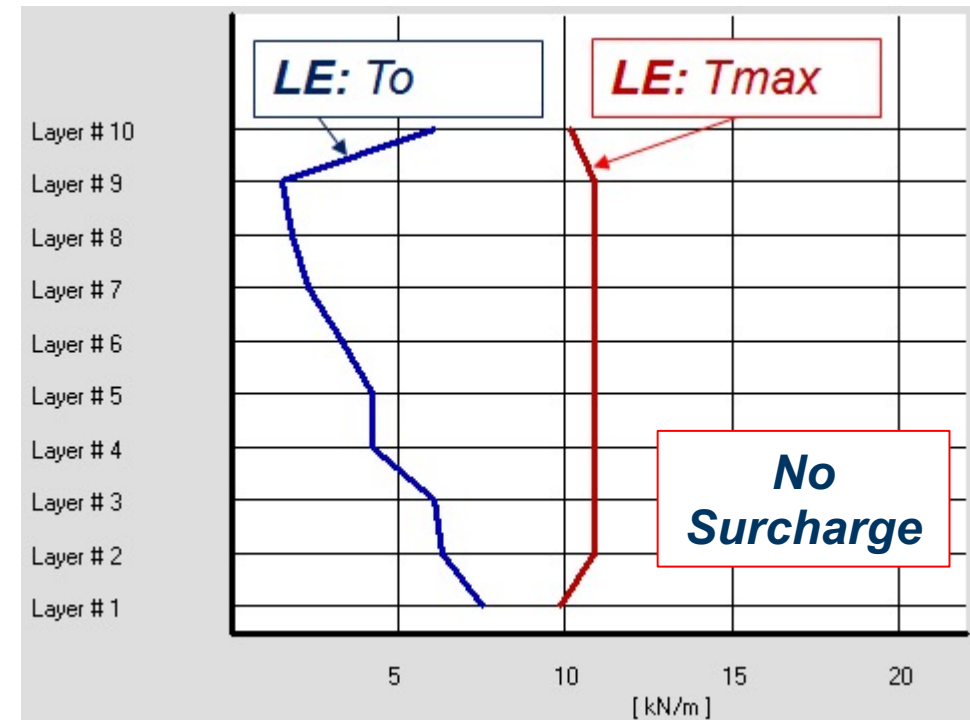
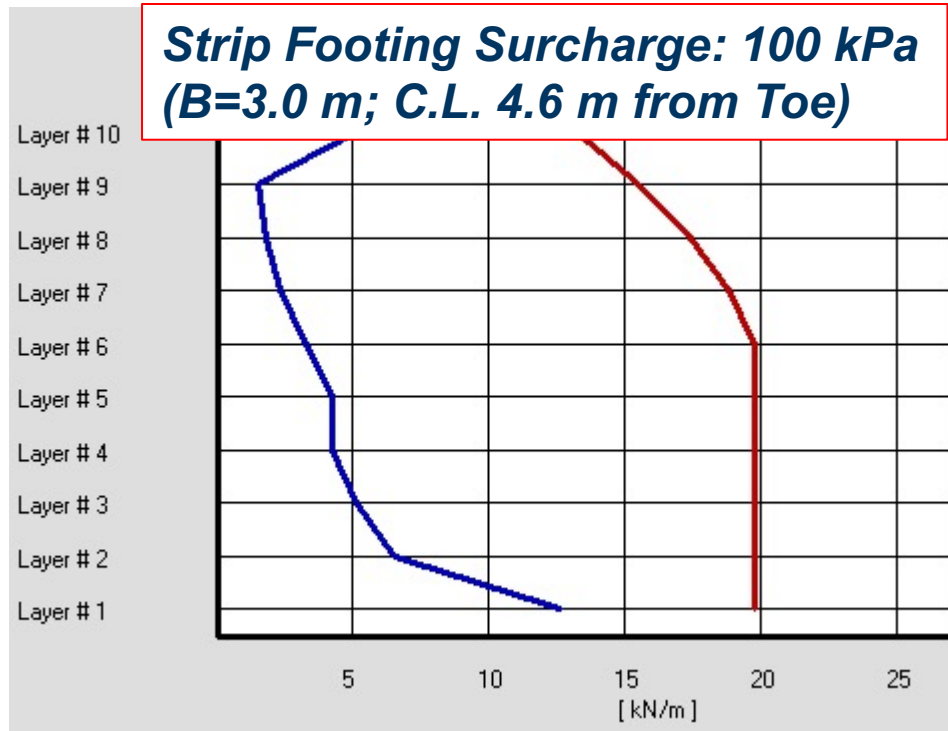
T_{max} and T_o



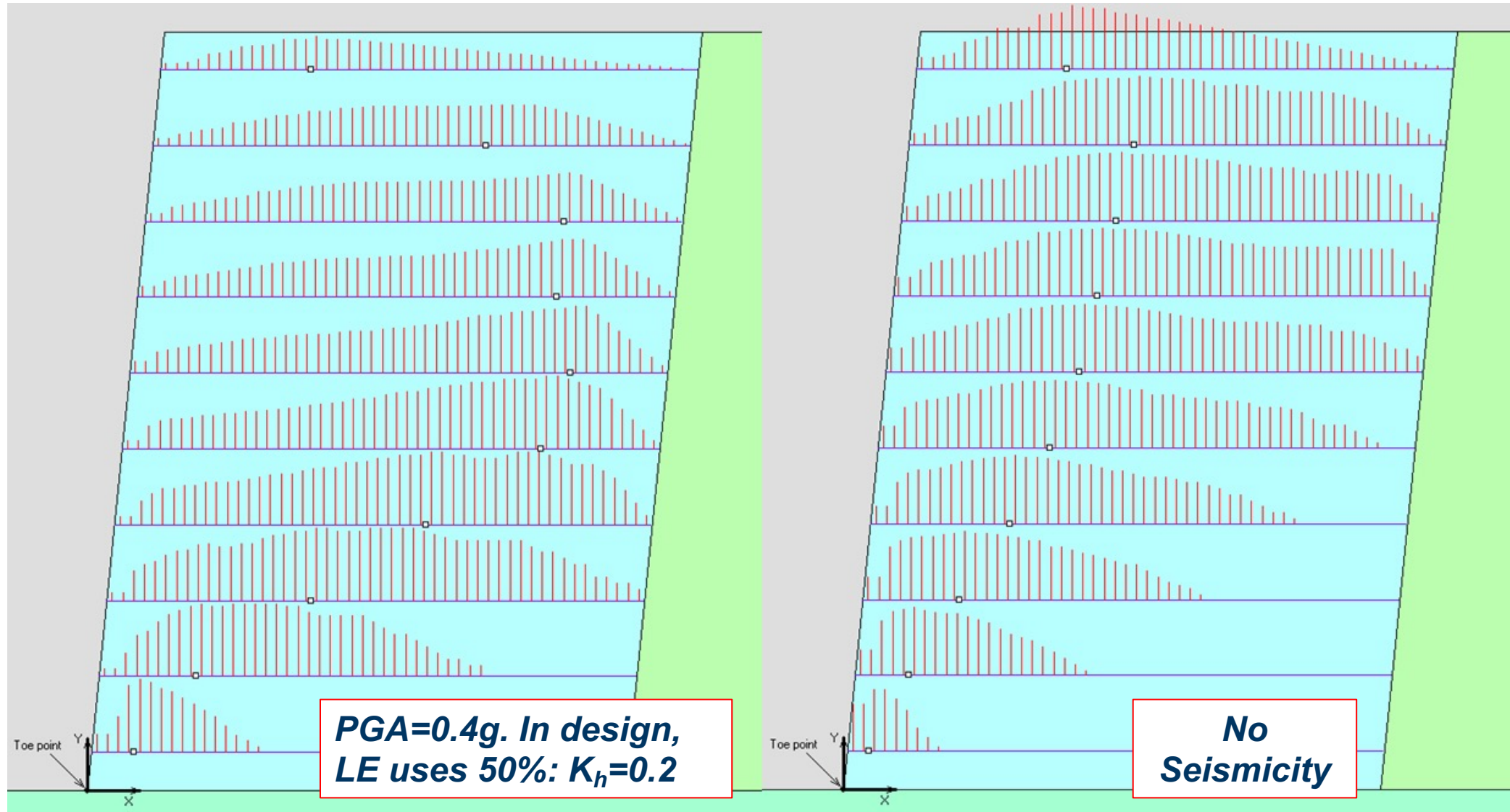
Effects of Surcharge (Dead Load)



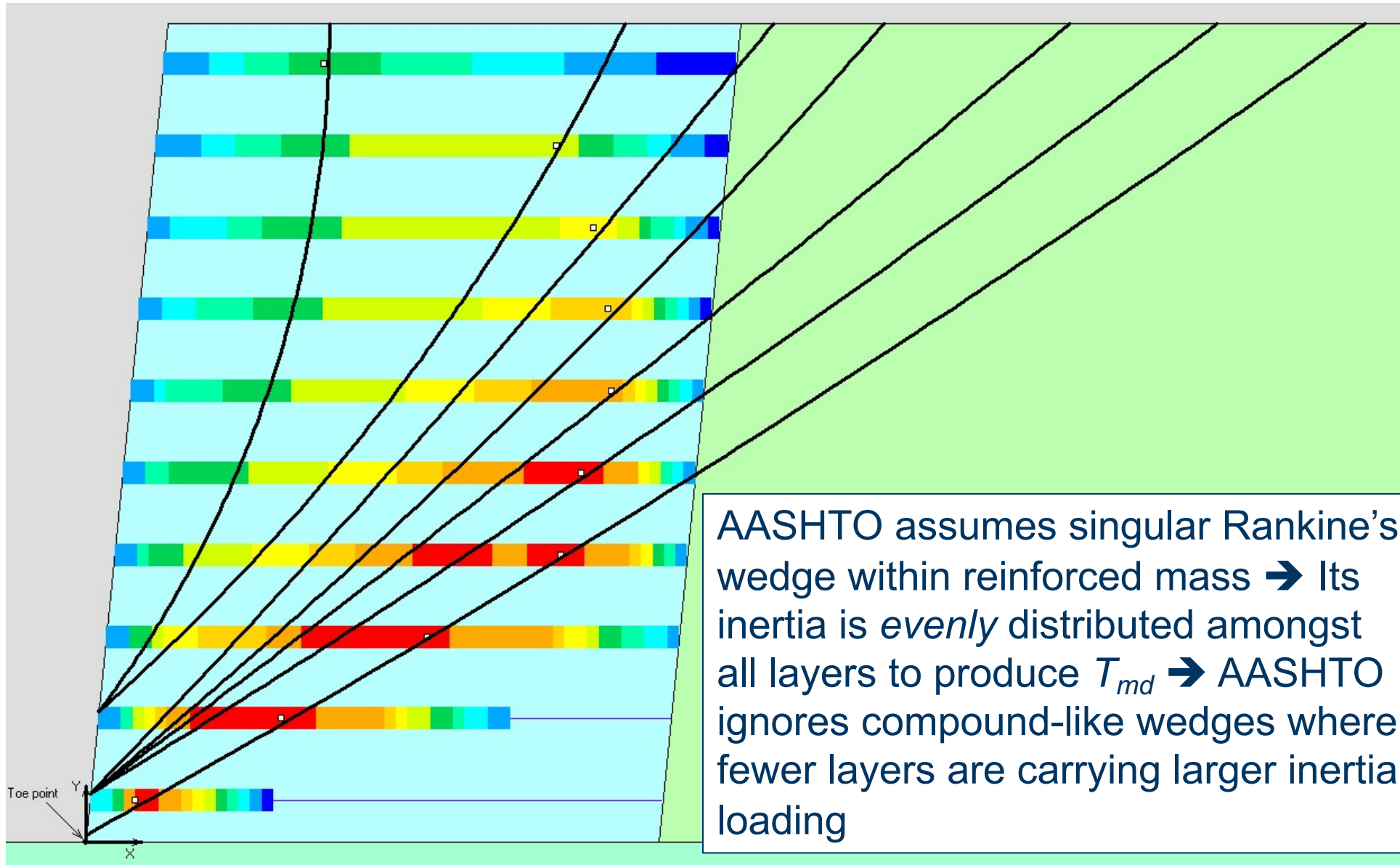
Effects of Surcharge: T_o and T_{max}



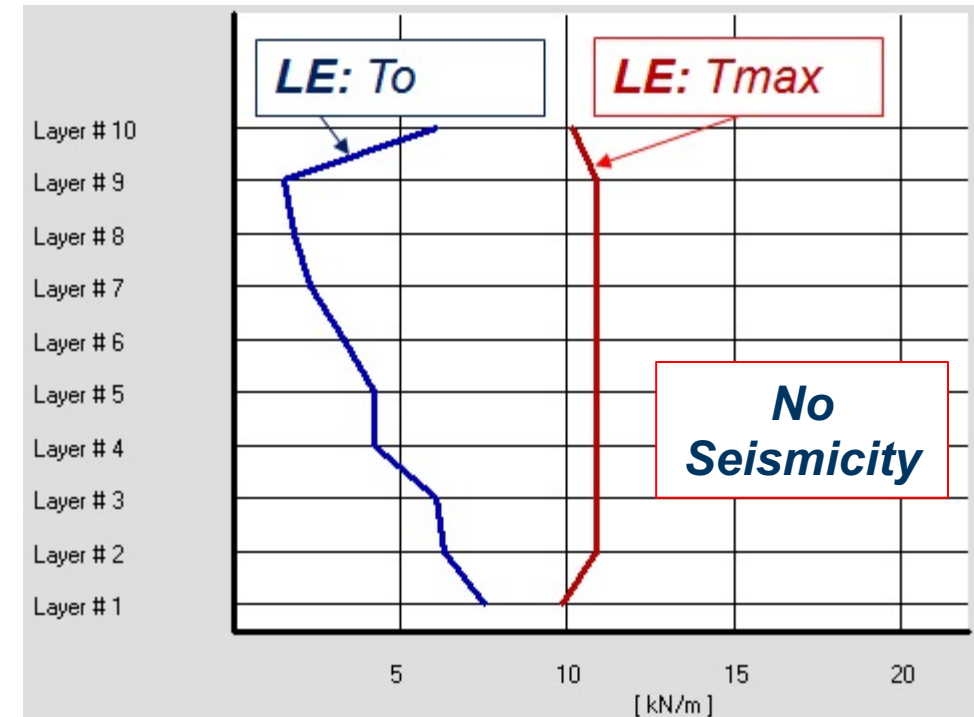
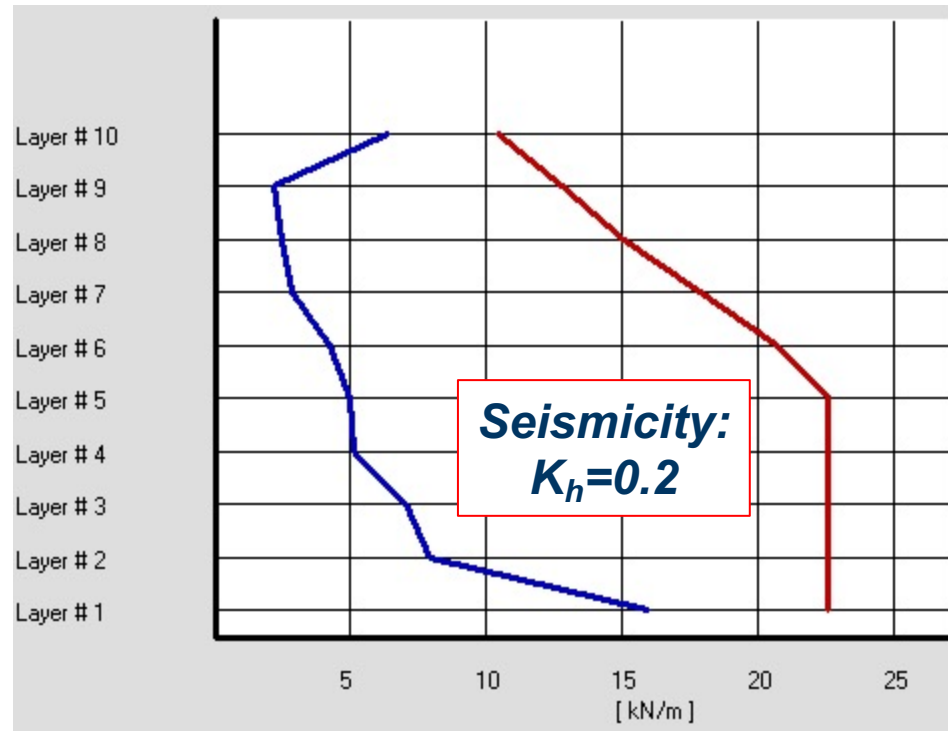
Effects of Seismicity



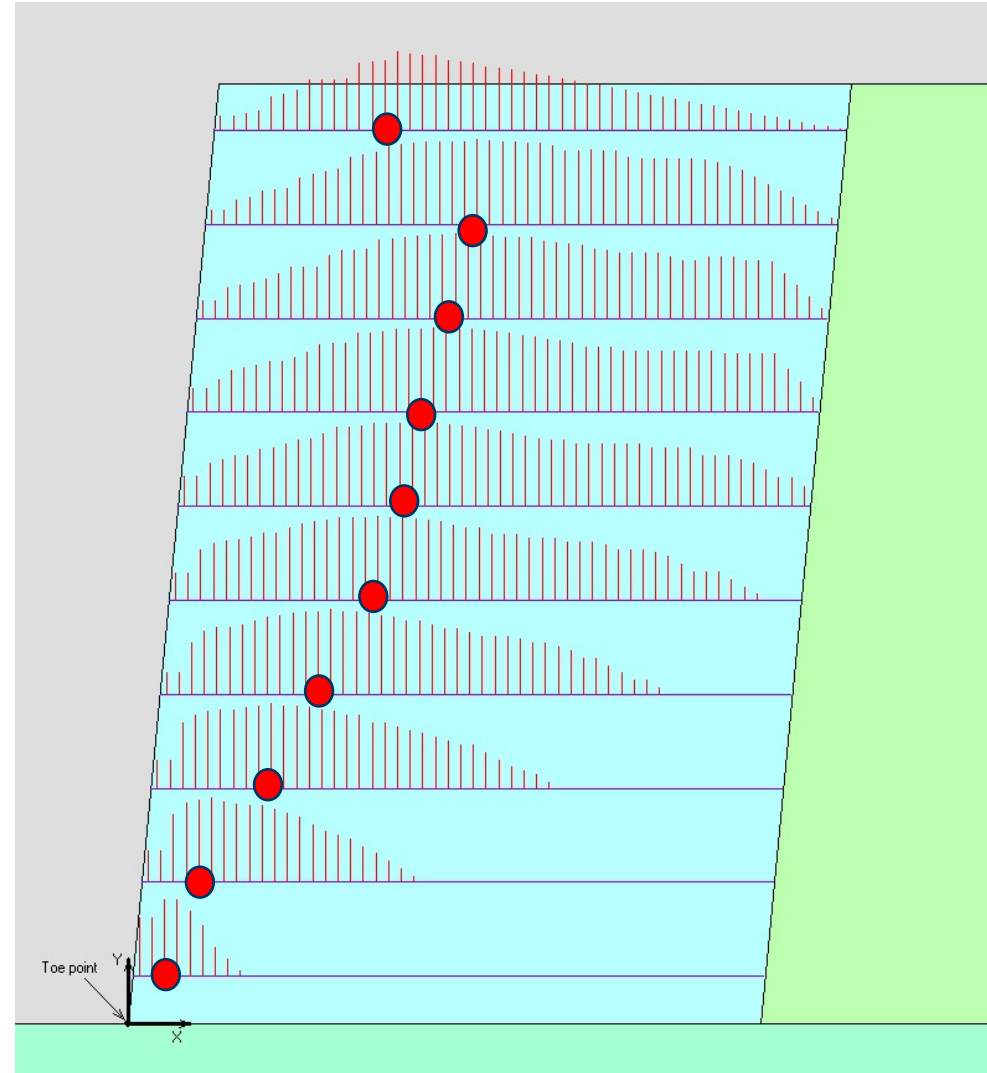
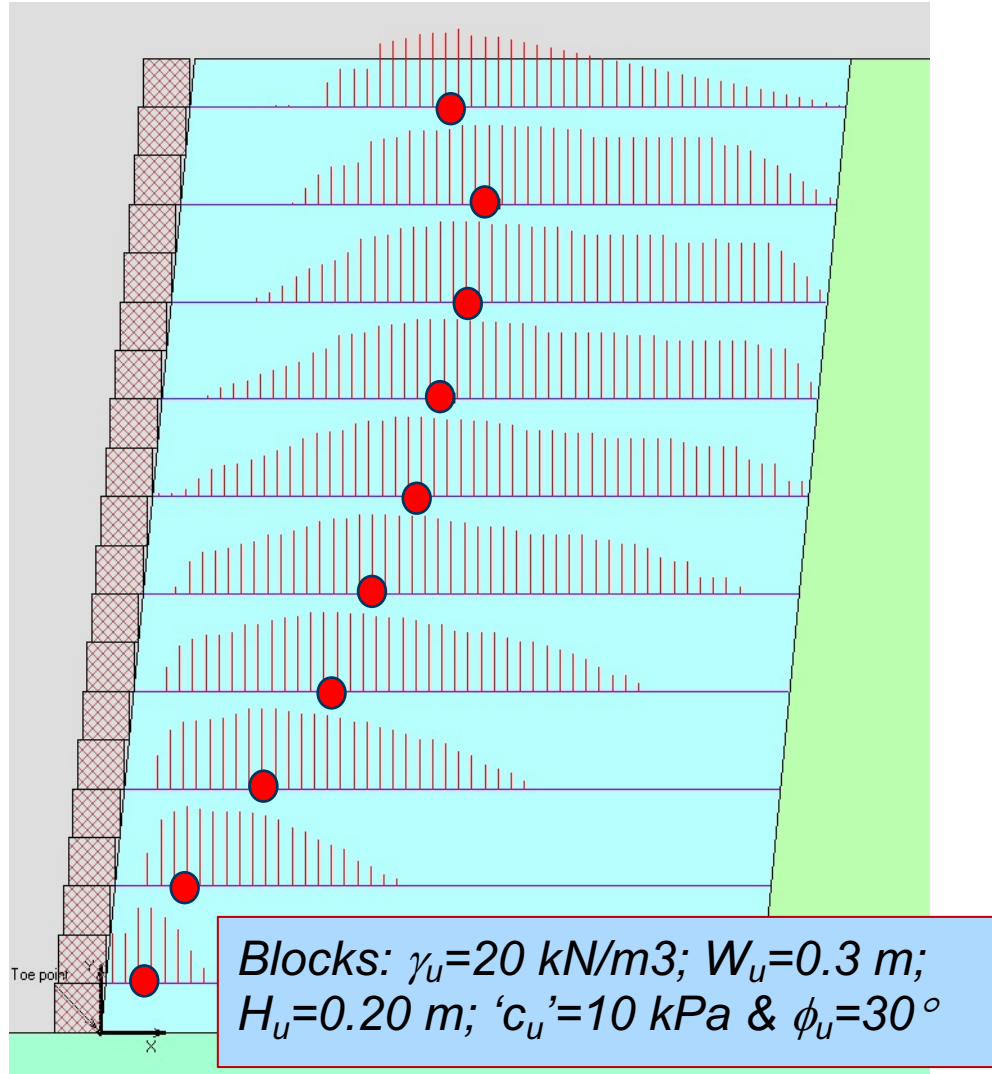
Critical Circles Rendering T_{\max}



Seismic Effects: T_o and T_{max}



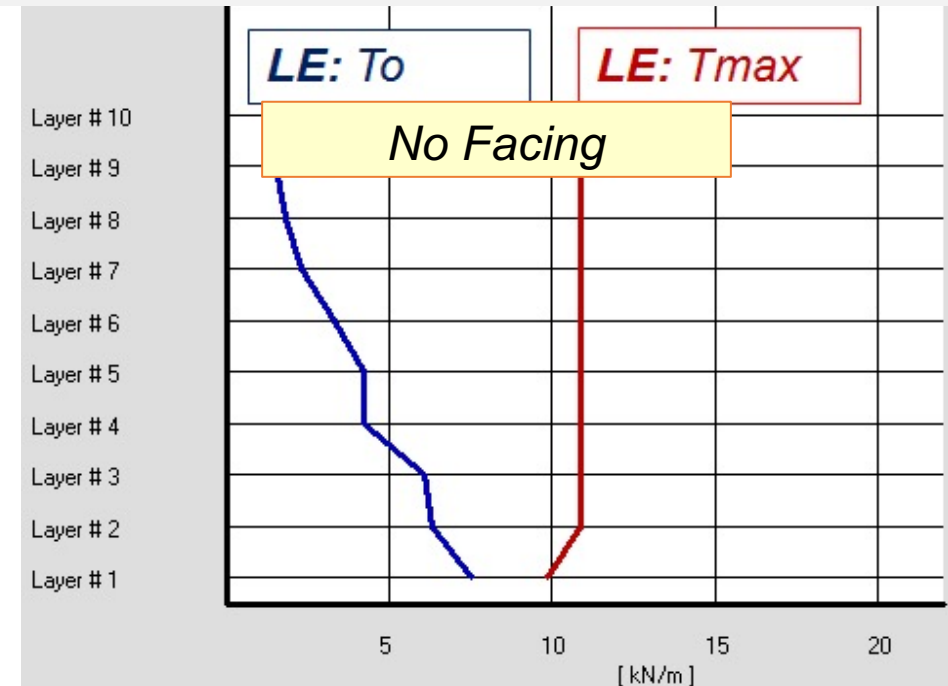
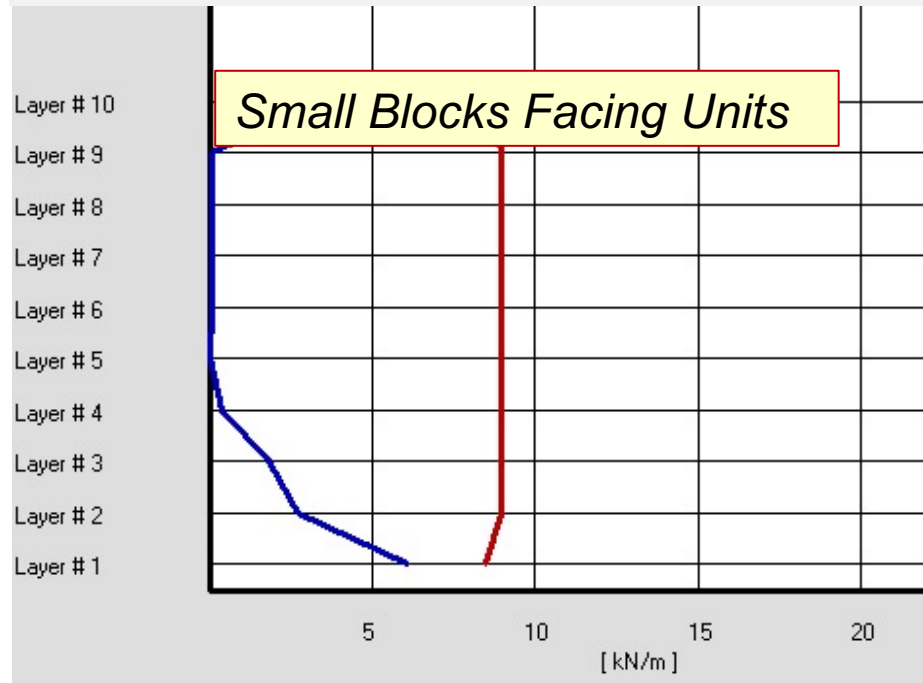
Effects of Facing: **Small Blocks**



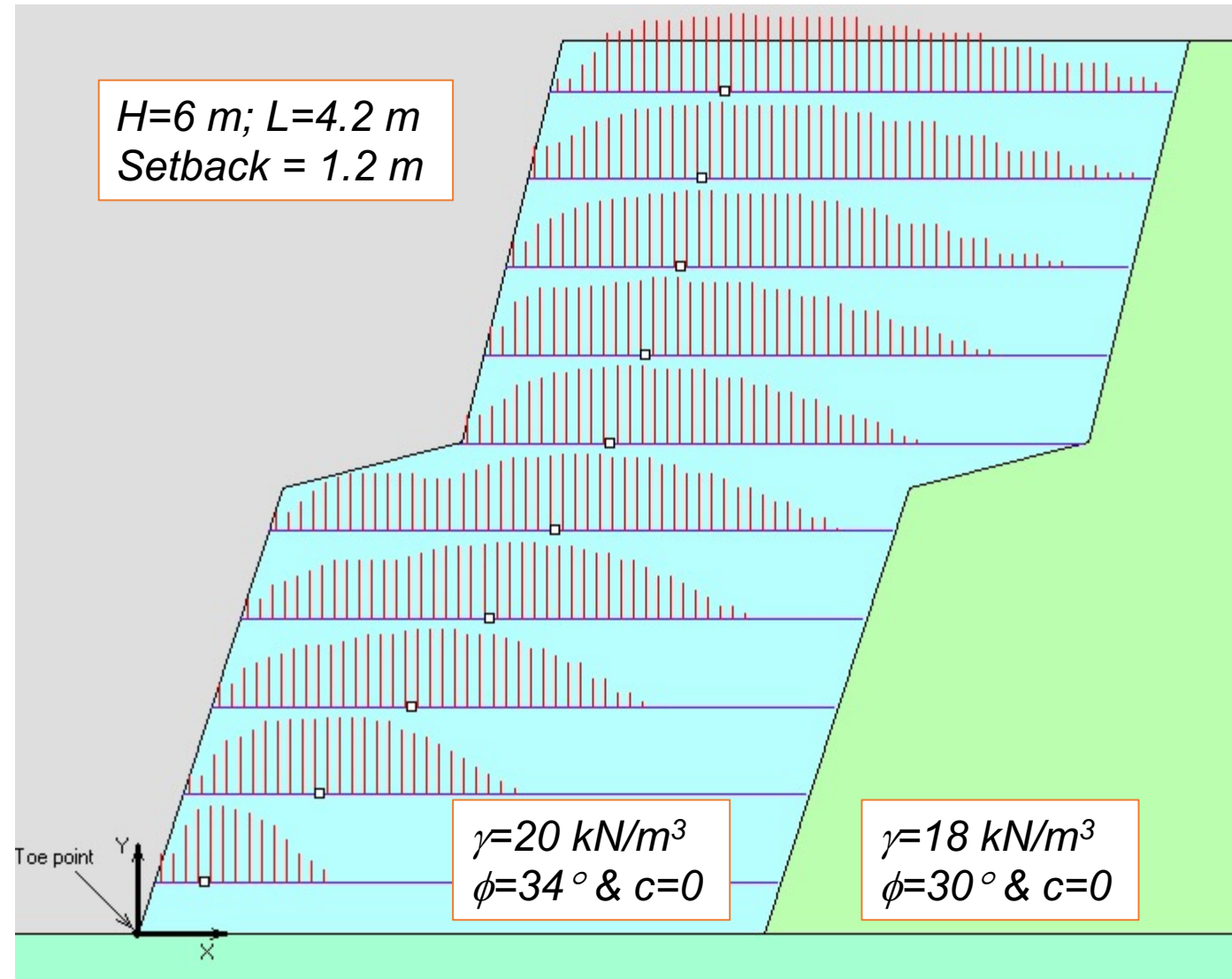
Effects of Small Blocks Facing:

T_{max} and T_o

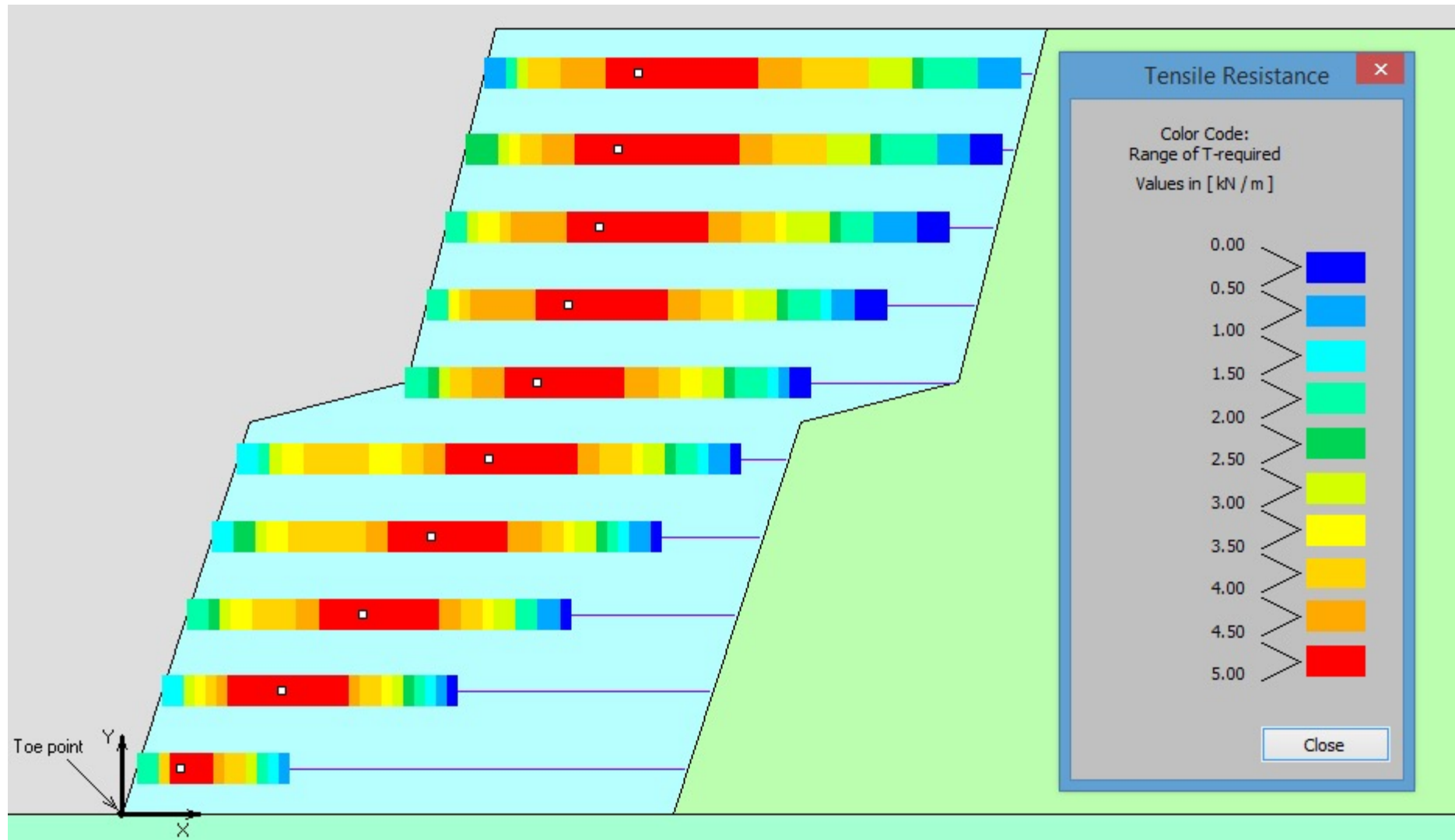
Large blocks or high interblock and toe resistance may reduce significantly the need for reinforcement (length and strength)



3(v):1(h) Two-Tier Wall



Tension Map: 2-Tier Wall



Roadmap of Presentation

- Why Limit State analysis is needed?
- Available Limit State Methods of Analysis
- Limit Equilibrium: Global Approach
- The Safety Map Tool
- Limit Equilibrium: Baseline Solution
- Limit Equilibrium: Design Approach
- Limit Equilibrium: Examples
- **Concluding Remarks**

Concluding Remarks

- ❑ Baseline Solution: $F_s=1.0$ on soil strength is used to determine LTDS, consistent with **Internal Stability** principles → LRFD can be used, same as in AASHTO
- ❑ T_{\max} and T_o : **Global Stability** ignores possible local overstressing while the **Baseline Solution** considers local demand rationally
- ❑ **Global LE**: Applicable to external stability -- sliding, eccentricity, and bearing load

Thank You!



小心跌落

Beware of Falling

དངོས་པོ་བྱགས་པར་སེམས་ཆུང་བྱེད།